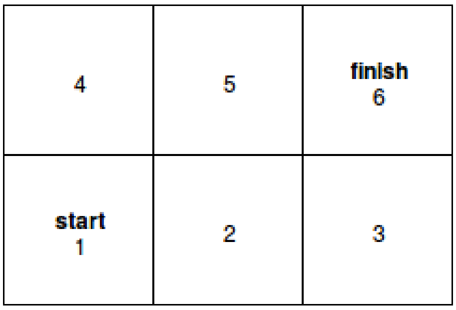
Answers Tutorial Reinforcement Learning

Course: Artificial Intelligence Techniques (IN4010)

Assume we are an agent in a 3x2 gridworld, as shown in the below figure. We start at the bottom left node (1) and finish in the top right node (6). When node 6 is reached, we receive a reward of +10 and we return to the start for a new episode. On all other actions that not lead to state 6, the reward is -1.



In each state we have four possible actions: up, down, left and right. For each action we move in the specific direction on the grid. However, there is always a 10% probability that we *slip*, which causes us to actually stay at the same location and not move at all (however, the reward is still -1). Assume that we cannot take actions that bring us outside the grid.

Question a Let $P^a_{ss'} = T(s, a, s')$ denote the probability of ending in state s' when taking a in s. Give T(2,right,3), T(2,right,2) and T(2,up,3).

Answer: 0.9 ; 0.1 and 0.

Assume our current policy is **random**. We can use Bellman's equation to update the values of each state under the current policy. Initialize all current $V(s)$ to 0. Bellman's equation is given by:

$$V^\pi(s) = \sum_a \pi(s,a) \sum_{s'} P^a_{ss'}[r^a_{ss'} + \gamma V^\pi(s')] \quad (1)$$

Question b Take discount parameter $\gamma = 0.5$. Update V(3) **once** according to Bellman. (Hint: be careful for which a is $\pi(3, a)$ positive).

Answer: Only two actions available, up and left, so for random policy $\pi(3, up) = 0.5$ *and* $\pi(3, left) = 0.5$. *Tricky aspect is the slipping which we again have to take into account.*

$V(3) = 0.5\big(0.9(-1+0.5\times 0)+0.1(-1+0.5\times 0)\big) \quad +0.5\big(0.9(10+0.5\times 0)+0.1(-1+0.5\times 0)\big) = -0.5+4.45 = 3.95$

John suggests we should not assume a model of the environment. He proposes to use a sampling based approach. In particular, he wants to use Q-learning, which implements the following one step update:

$$Q(s,a) = Q(s,a) + \alpha[r_{sas'} + \gamma \max_b Q(s',b) - Q(s,a)] \tag{2}$$

John has already made some steps in this process. He gives you the following table with his current estimates:

Q(1,up)=3	Q(1,down)=.	Q(1,left)=.	Q(1,right)=5
Q(2,up)=5	Q(2,down)=.	Q(2,left)=2	Q(2,right)=6
Q(3,up)=8	Q(3,down)=.	Q(3,left)=3	Q(3,right)=.
Q(4,up)=.	Q(4,down)=2	Q(4,left)=.	Q(4,right)=4
Q(5,up)=.	Q(5,down)=1	Q(5,left)=3	Q(5,right)=7

IMPORTANT!: From now on assume there is no more slipping, i.e. each actions leads deterministically to the next node. So for example, taking action right in node 2 always brings you in node 3.

Question c What is the Q-value for node 6, for example: what is Q(6,down)?

Answer: Node 6 is terminal (no outgoing links), so its value is by definition 0 for each action.

Question d Imagine we start exploitation now, i.e. we take a greedy policy. What policy will the agent follow from the start node. You can indicate the trajectory. Write down the equation you base your greedy choice on.

Answer: Equation for the greedy policy:

$$\pi(s) = \arg\max_a Q(s,a) \tag{3}$$

This results in the trajectory: state 1 - state 2 - state 3 - state 6. Or in the actions: right - right - up. (Both are correct).

Note that we officially write $\pi(s,a)$ for the policy, and it returns a probability distribution of actions (i.e. $\sum_a \pi(s,a) = 1$). However, $\pi(s)$ is shorthand for a deterministic greedy policy, and it returns the action to which it assigns probability 1.

Question e John goes to lunch and asks you to continue his work. He says he stopped in state 4 and uses an $\epsilon - greedy$ exploration policy with $\epsilon = 0.20$. He has been drawing random numbers for each step: if the number is smaller than 0.20 he makes an exploring step (excluding the greedy action). Else, he follows the greedy action. The two next numbers are: 0.14 and 0.70. Make the two next updates following Q- learning with $\alpha = 0.1$ and $\gamma = 0.5$. For each step, fill in the form and calculate the update.

Answer: first step explore, so take action down from state 4 to state 1. Second step exploit, so take action right from state 1 to state 2. Plugging in from the Q-table into equation 2 gives:

s	*a*	*r*	*s'*
4	*down*	*-1*	*1*

$Q(4, down) = 2 + 0.1(-1 + 0.5 \times 5 - 2) = 2 - 0.05 = 1.95$

s	*a*	*r*	*s'*
1	*right*	*-1*	*2*

$Q(1, right) = 5 + 0.1(-1 + 0.5 \times 6 - 5) = 4.7$