Sample-based Planning

Introduction To Reinforcement Learning, Leiden University, The Netherlands

Thomas Moerland

Planning/Search

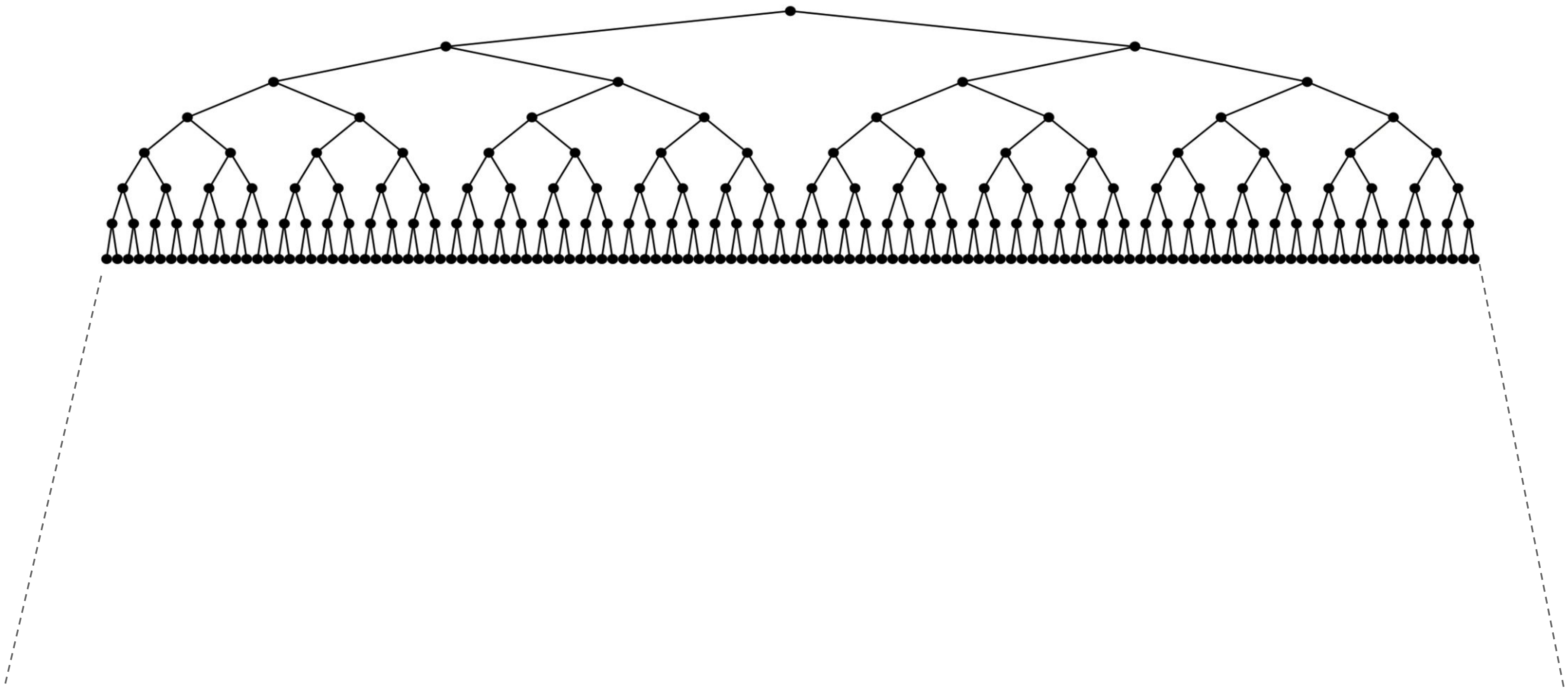
Planning/Search

Definition?

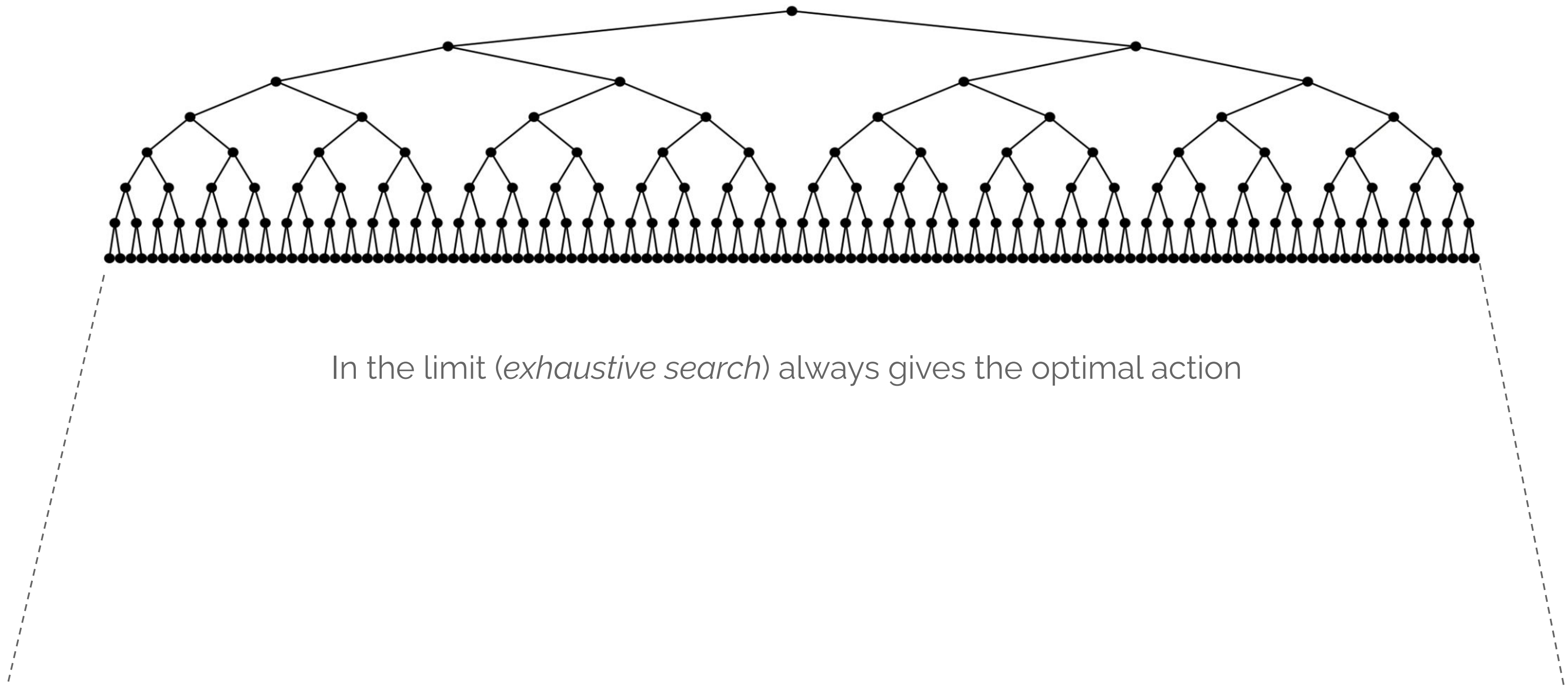
Planning/Search

Any type of lookahead search in a model to determine good actions

Planning/Search

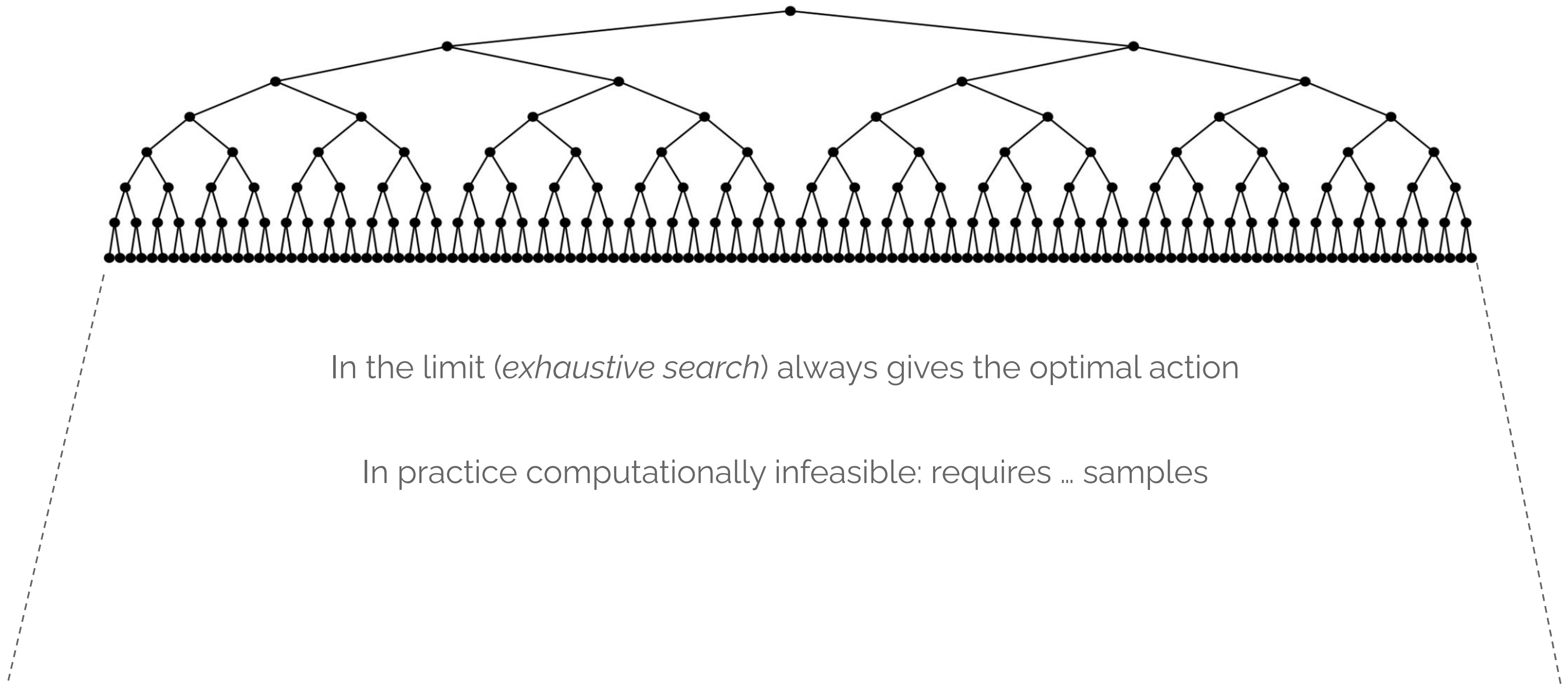


Planning/Search



In the limit (*exhaustive search*) always gives the optimal action

Planning/Search



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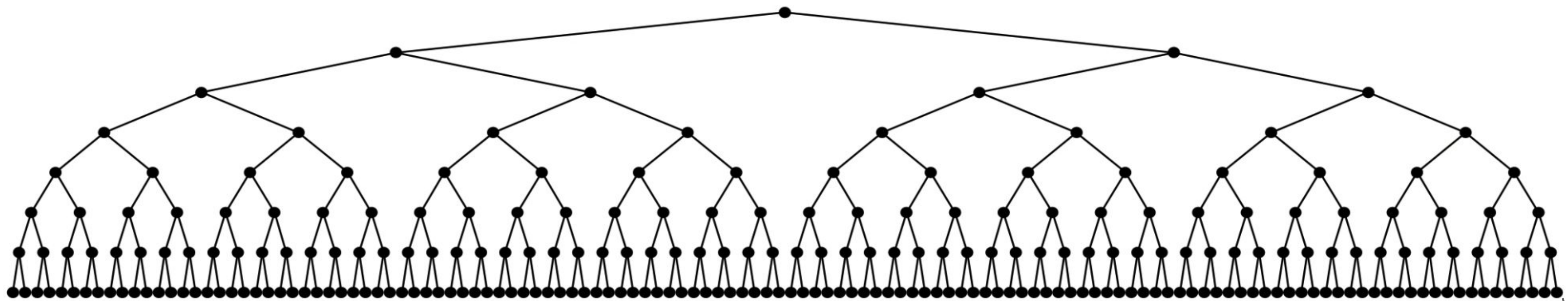
In practice computationally infeasible: requires … samples

Planning/Search

b=branching factor (# actions)

d= depth

(we for now ignore stochastic transitions)



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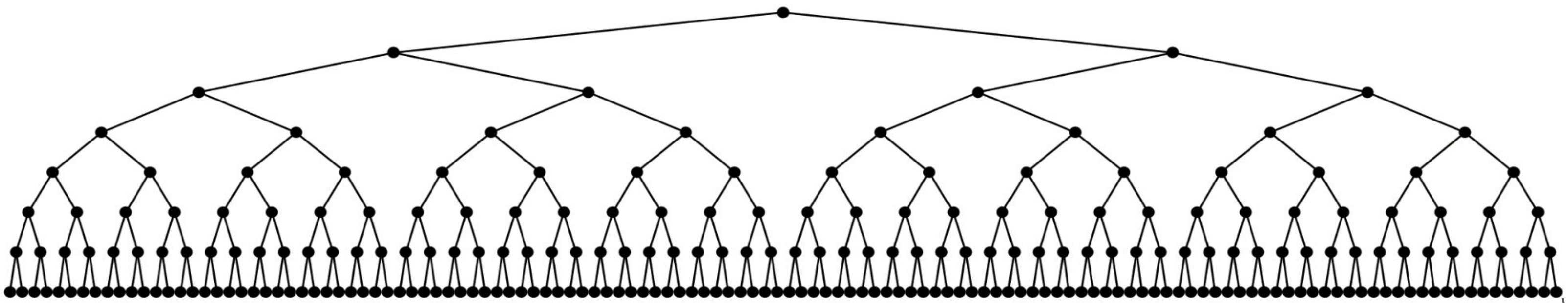

In the limit (*exhaustive search*) always gives the optimal action

In practice computationally infeasible: requires $\mathbf{b}^{\mathbf{d}}$ samples

All search algorithms try to improve the visitation order
(i.e., reduce the width and depth of the search)

Content

1. Types of planning (decision-time versus background)

2. Classic planning (uninformed & heuristic search)

Break

3. Sample-based planning (Monte Carlo Search, Sparse Sampling, Monte Carlo Tree Search)

4. Iterated planning and learning

1. Decision-time versus Background planning

Type of planning

Type of planning

Background planning

Decision-time planning

Type of planning

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Use lookahead in model to update a global (value/policy) solution

(improve overall solution – may be called 'learning')

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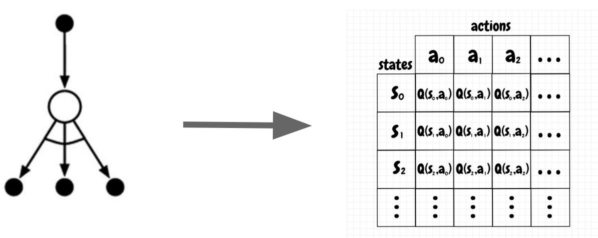

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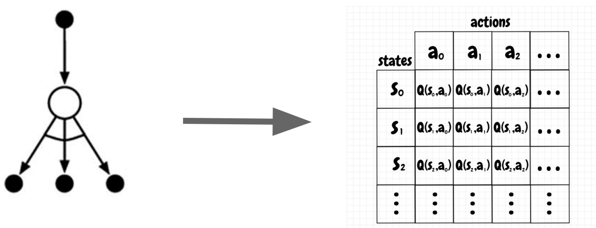

e.g. Dynamic Prog. (Ch. 4), Dyna (Ch. 8)

(Traditionally: smaller tree)

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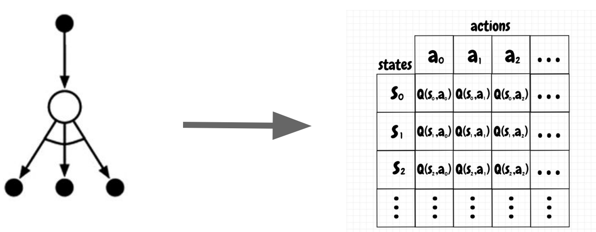

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Decision-time planning

Use lookahead in model to find a good action for a current state s

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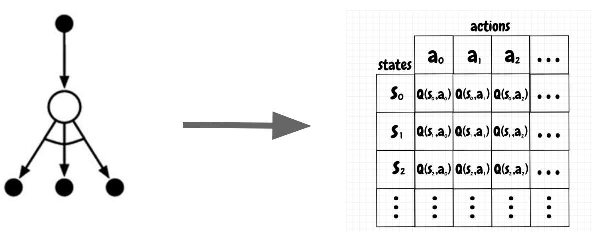

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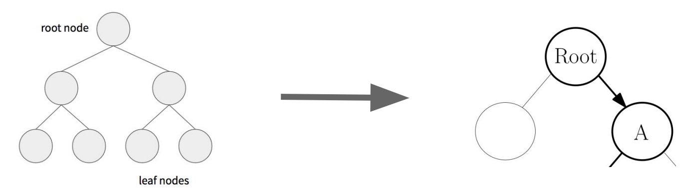

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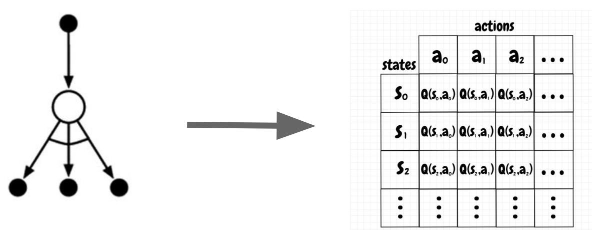

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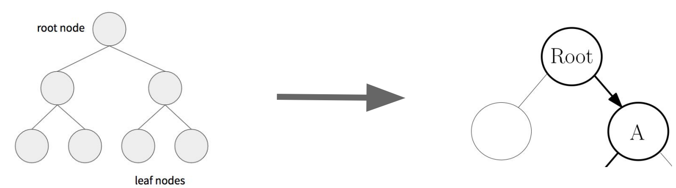


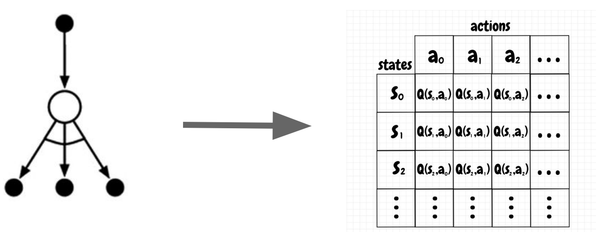
e.g. A*, MCTS (Ch. 8)

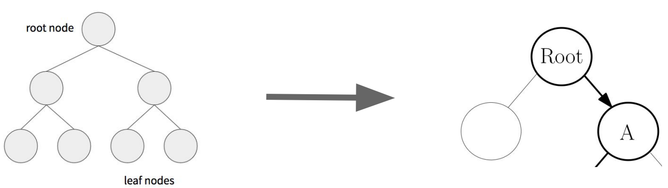
(Traditionally: larger tree, afterward discarded)

Type of planning

Main topic of today

Background planning	Decision-time planning
Use lookahead in model to update a global (value/policy) solution	Use lookahead in model to find a good action for a current state s
(improve overall solution – may be called ‘learning’)	(focus all budget on current decision)





e.g. Dynamic Prog. (Ch. 4), Dyna (Ch. 8)	e.g. A*, MCTS (Ch. 8)
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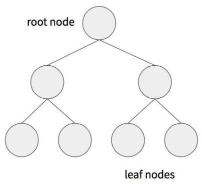
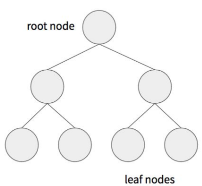


Arbitrarily small/large search

Both planning types can be combined



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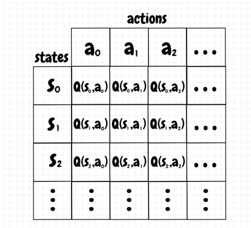
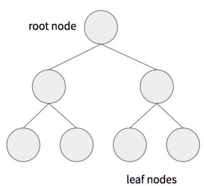


1. update overall solution (background/learning)

Both planning types can be combined



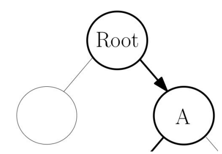
Arbitrarily small/large search

actions

states	a_0	a_1	a_2	...
s_0	$Q(s_0,a_0)$	$Q(s_0,a_1)$	$Q(s_0,a_2)$	...
s_1	$Q(s_1,a_0)$	$Q(s_1,a_1)$	$Q(s_1,a_2)$	...
s_2	$Q(s_2,a_0)$	$Q(s_2,a_1)$	$Q(s_2,a_2)$	...
⋮	⋮	⋮	⋮	⋮

1. update overall solution (background/learning)

and/or



2. select an action (decision-time)

Both planning types can be combined

may also revert

		actions		
states	a_0	a_1	a_2	...
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⋮	⋮	⋮	⋮	⋮

1. update overall solution
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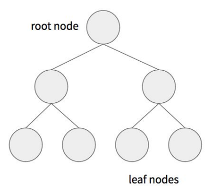
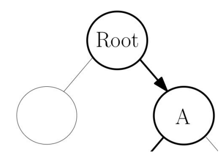


and/or

Arbitrarily small/large search



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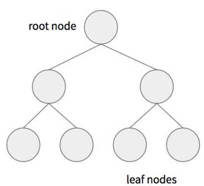

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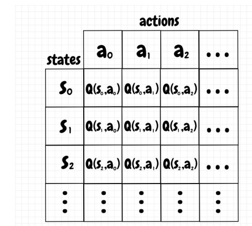

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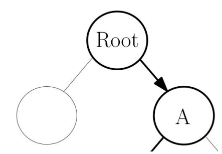


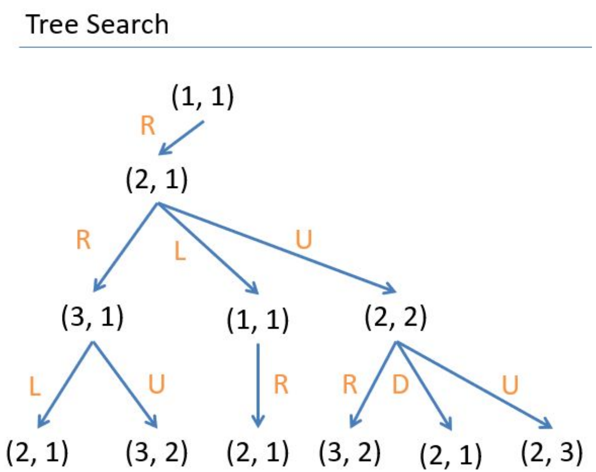
2. select an action (decision-time)

Discuss combined combined planning
& learning at end of this lecture

2. Classic Planning

Tree versus graph search

Tree versus graph search



Tree versus graph search

Q: Can a tree search spend useless compute?

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A: Yes, because the same next state may appear in multiple directions

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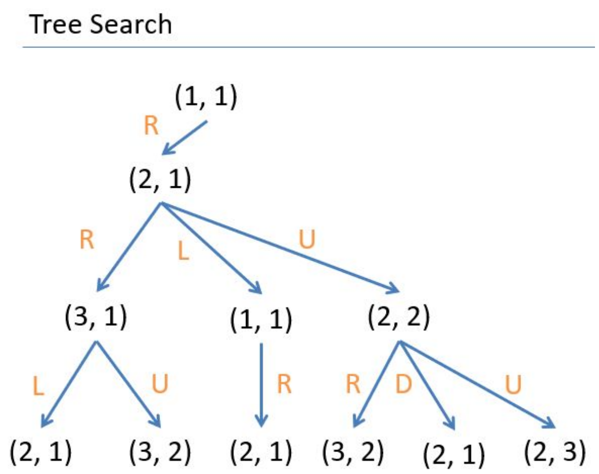
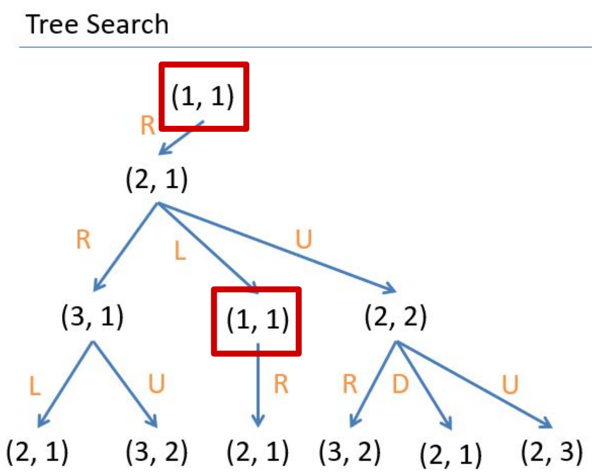
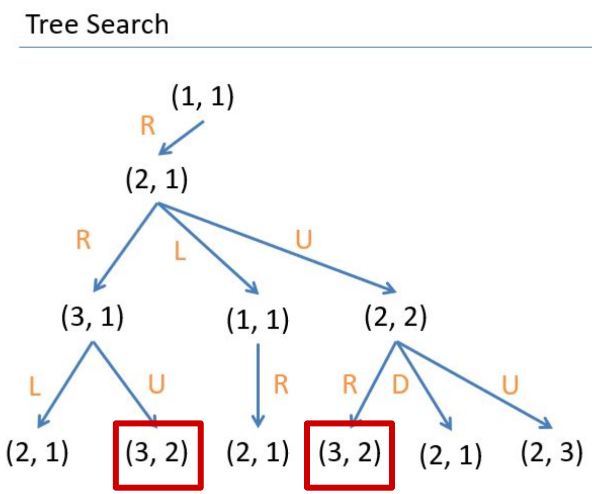


Tree versus graph search



'loop' : same state reappears in a path → only need to search from the first appearance

Tree versus graph search



redundant path: same state appears in different arms → only need to continue the search in the best path

Tree versus graph search

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Q: What could be a solution?

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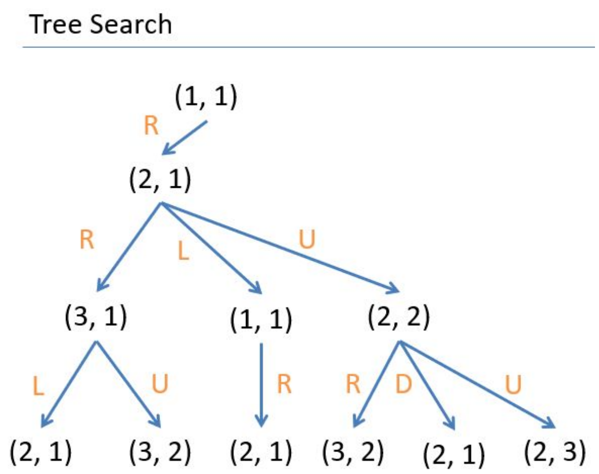

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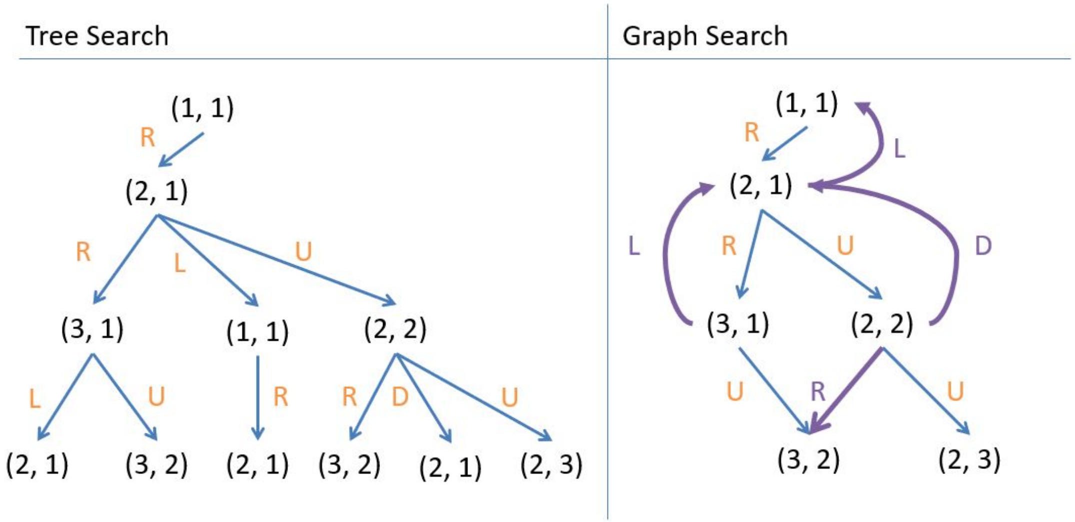

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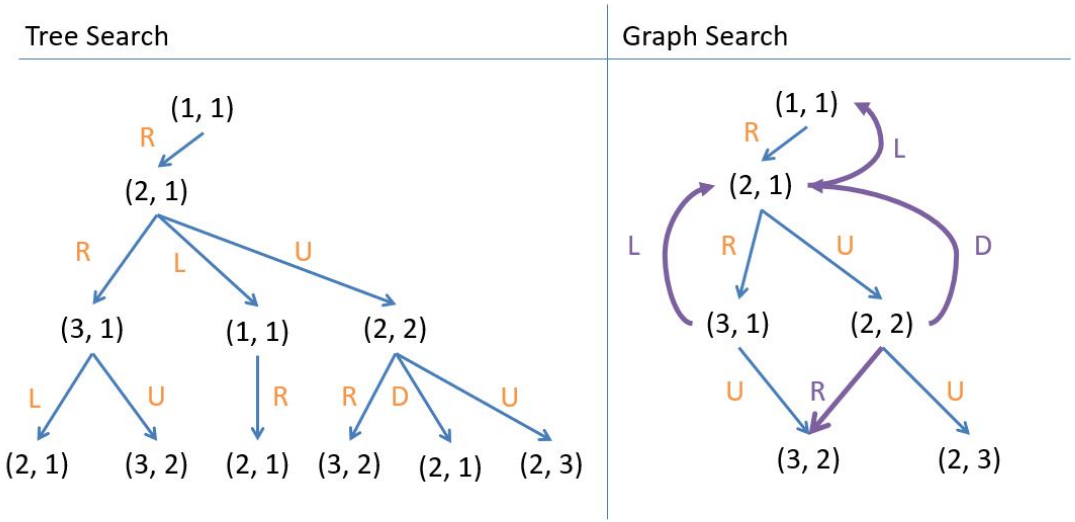

Build a **graph**: only generate each unique state once, and build a search tree connecting them

Tree versus graph search

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Q: What could be a solution?

A: Turn the tree search into a **graph search**

Q: What do we need to store/change for this?

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A: Turn the tree search into a **graph search**

Q: What do we need to store/change for this?

A: Track an **open list** (*frontier*) and **closed list** *(explored set)*

Closed & Open list

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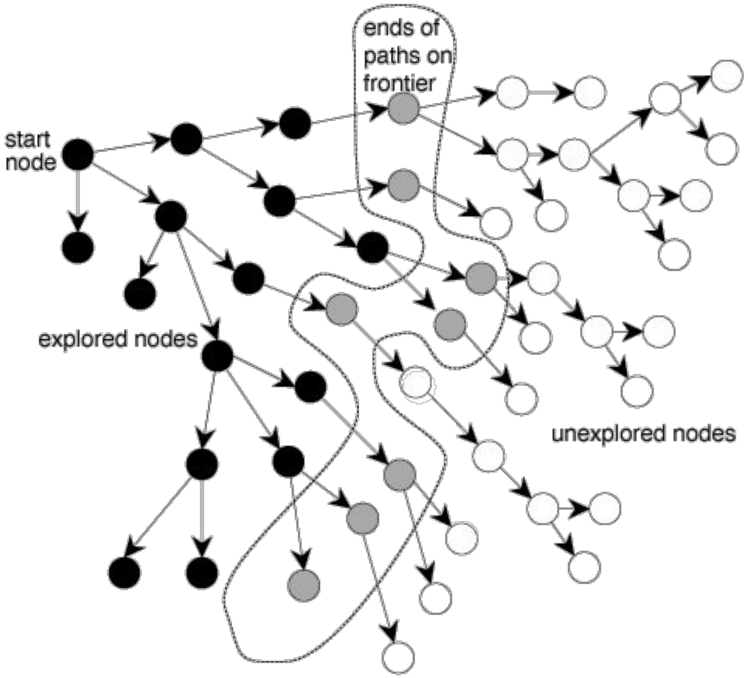
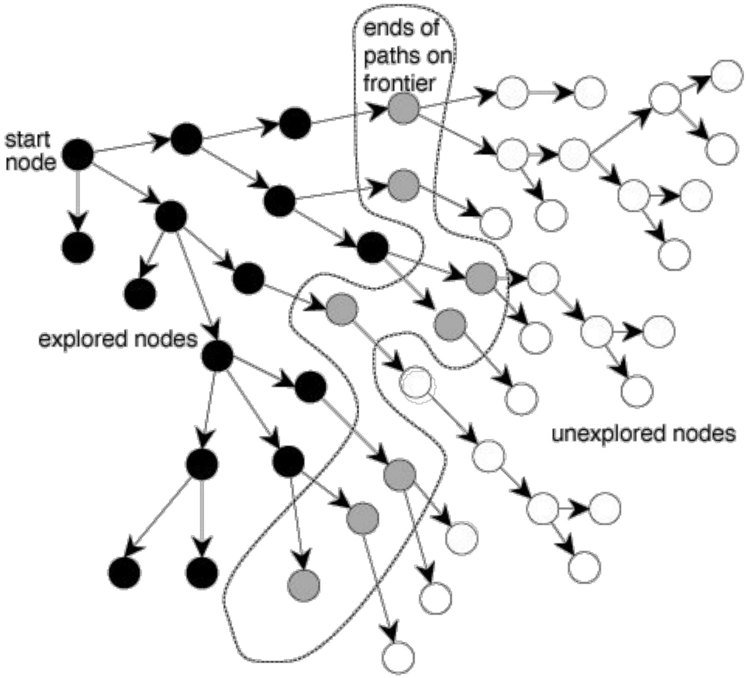


Closed & Open list

1. **Closed list**

   Fully expanded



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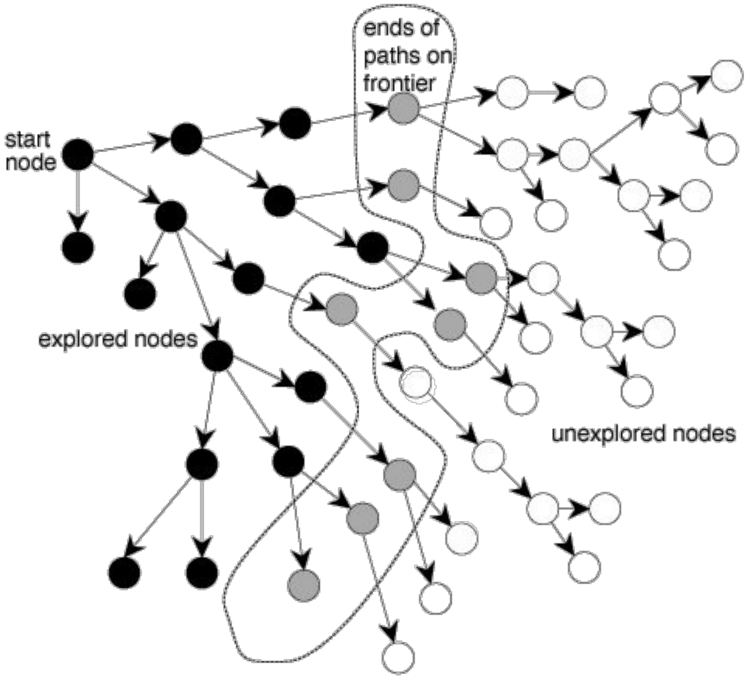
2. **Open list = Frontier**

   Next candidates for expansion



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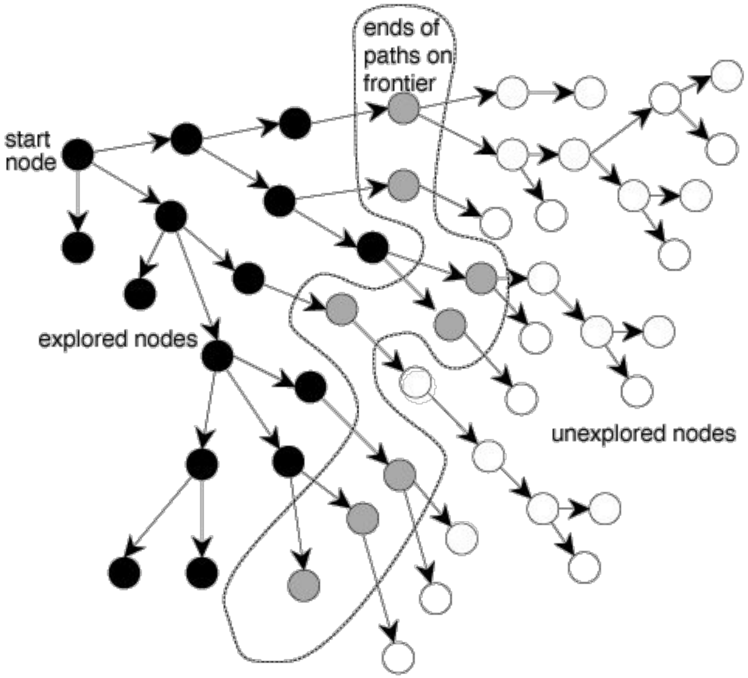
<u>Key idea</u>:

- Track every node in the graph (open/closed) and the optimal path towards is



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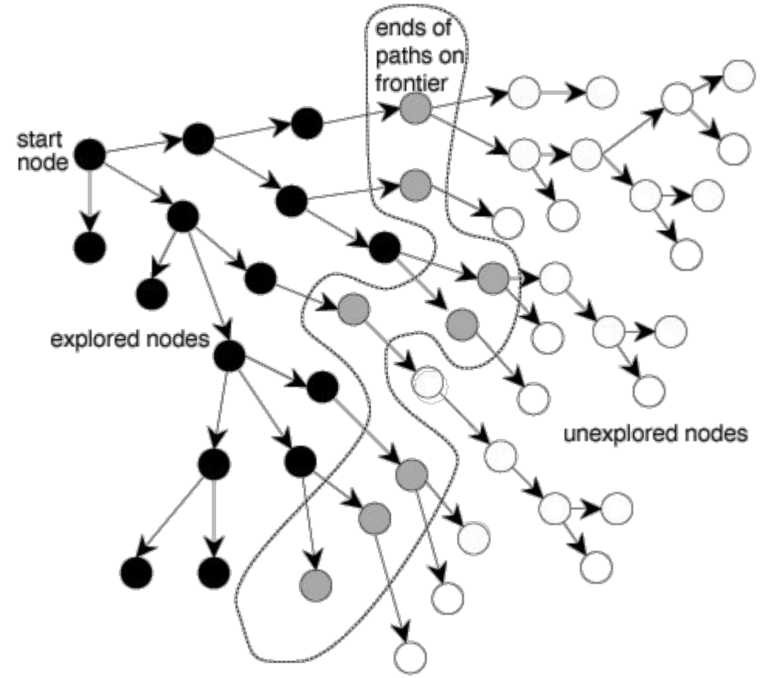

<u>Key idea</u>:

- Track every node in the graph (open/closed) and the optimal path towards is
- Update these lists with every expansion (is this new expansion already in my closed or open list?)

Main challenge of planning

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In what order shall we visit state-actions?

Uninformed search

Can you give some example of uninformed search strategies?

Uninformed search

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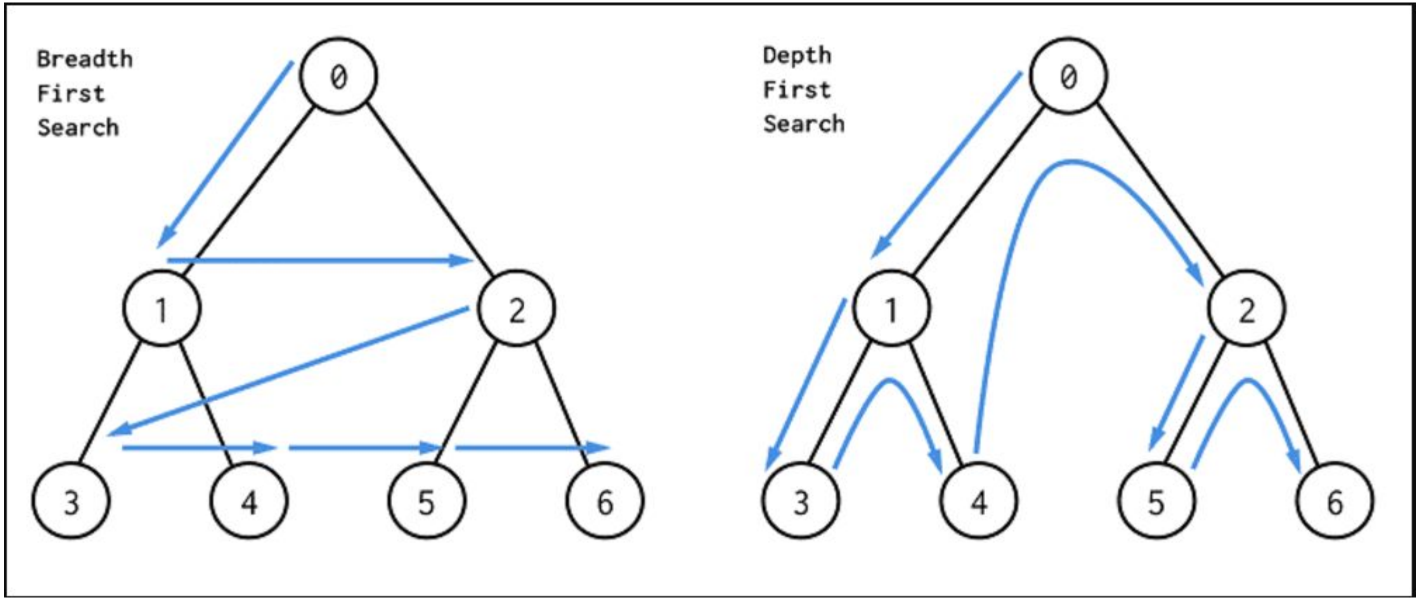
- Breadth-first search
- Depth-first search
- Iterative deepening
- Uniform cost search / Dijkstra’s algorithm (weighted graphs)

Uninformed search



Uninformed search

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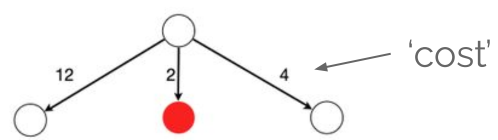

(BFS would expand the first action, but the second has lowest cost)

Reward versus Cost

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Reinforcement learning

Maximize the cumulative *reward*

Planning

Minimize the cumulative *cost*

Reward versus Cost

Reinforcement learning		**Planning**
Maximize the cumulative *reward*	=	*Minimize* the cumulative *cost*

same formulation

(cost = negative reward)

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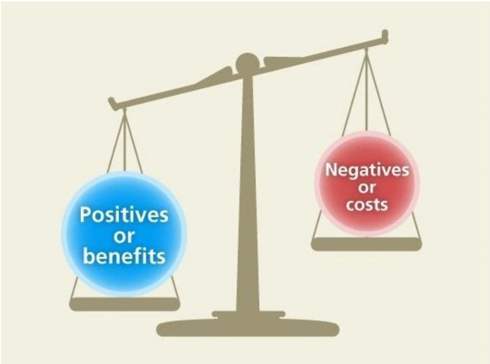

Reward versus Cost

Reinforcement learning		**Planning**
Maximize the cumulative *reward*	=	*Minimize* the cumulative *cost*

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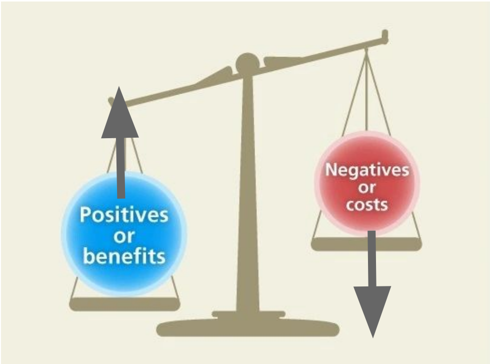

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What is a potential solution?

Uninformed search

What is the downside of breadth/depth first search?

- Can be suboptimal if the weight (*reward/cost*) per edge varies (ignored by DFS/BFS)

What is a potential solution?

- Expand the node which currently looks most promising

Uniform cost search / Dijkstra's algorithm

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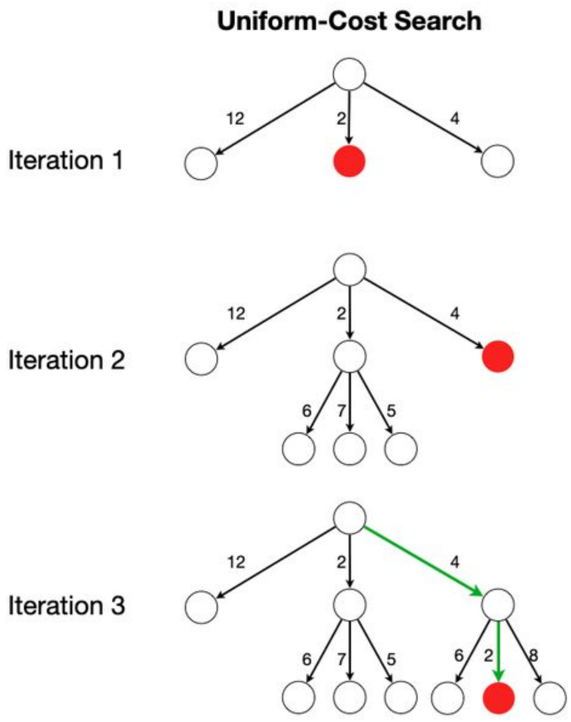
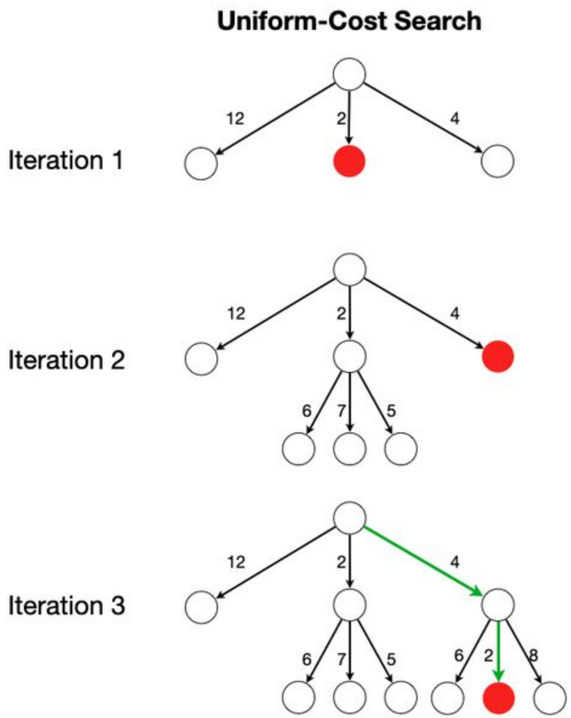


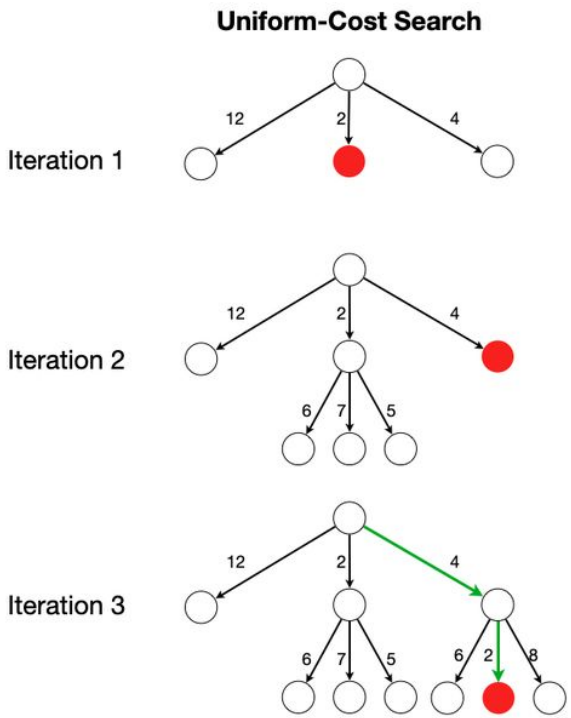
Uniform cost search / Dijkstra's algorithm



Note: only useful with non-uniform rewards/cost/edge weights

Q: What does this reduce to with uniform rewards?

Uniform cost search / Dijkstra's algorithm



Note: only useful with non-uniform rewards/cost/edge weights

Q: What does this reduce to with uniform rewards?
A: Breadth-first search

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Uniform cost search / Dijkstra's algorithm

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What is a potential solution?

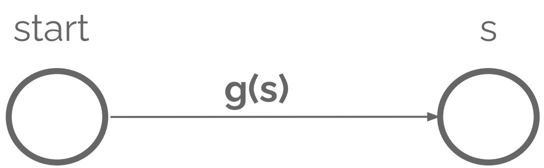
- Construct a heuristic function h(s) to predict the remaining potential

g(s) & h(s)

g(s) & h(s)

g(s)

g(s) = actual cumulative cost from start to state s

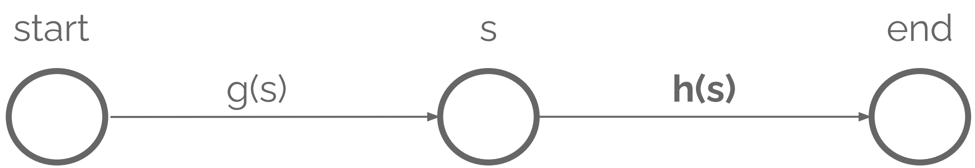


g(s) & h(s)

g(s) + **h(s)**

g(s) = actual cumulative cost from start to state s

h(s) = estimated cumulative cost from s to end



Best-first search

Best-first search

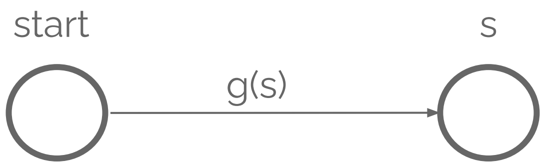
We want a general prioritization function **f(s)** to indicate what state to expand next

Best-first search

Best-first search

$f(s) = g(s)$

Dijkstra's algorithm/Uniform-cost search

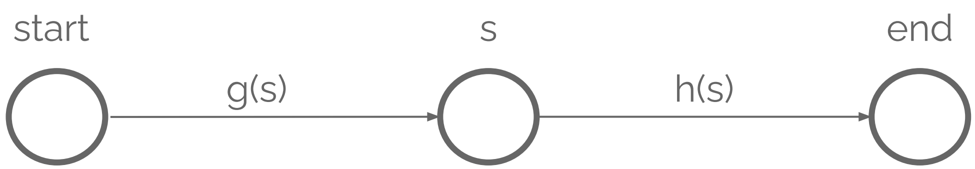


Best-first search

$f(s) = g(s)$	Dijkstra's algorithm/Uniform-cost search
$f(s) = h(s)$	Greedy best-first search

Best-first search

$f(s) = g(s)$	Dijkstra's algorithm/Uniform-cost search
$f(s) = h(s)$	Greedy best-first search
$f(s) = g(s) + h(s)$	**A* search**



A* search

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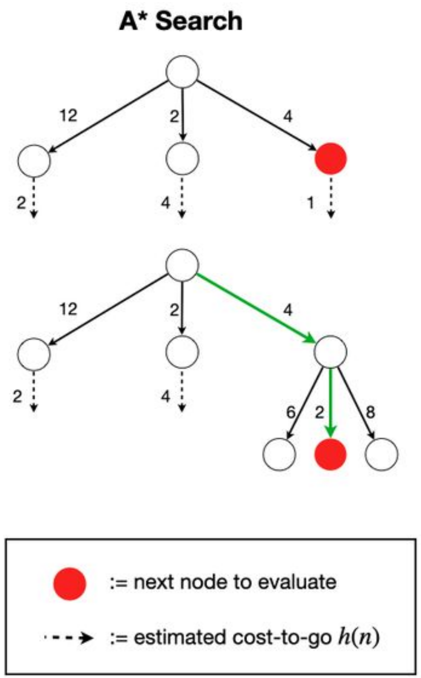
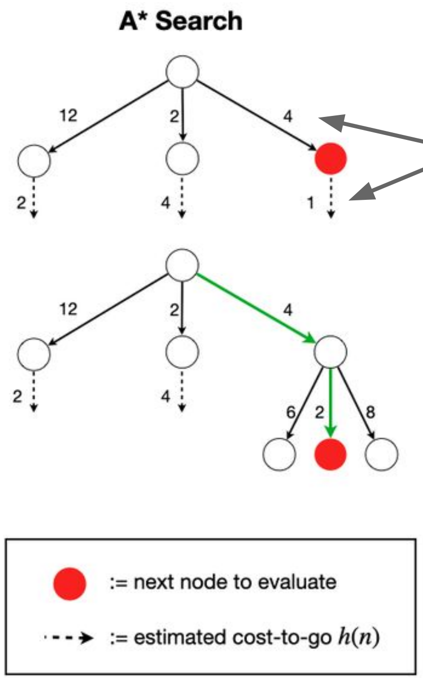

A* Search
12
2
4
2
4
1
12
2
4
2
4
6
2
8
:= next node to evaluate
:= estimated cost-to-go $h(n)$

A* search



Expand state with best g(s) + h(s)

Heuristic

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Heuristic function h(s) is typically obtained from prior domain-knowledge or relaxations

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Q: What is the perfect heuristic function?

A: The optimal value function: $h(s) = V^*(s)$! (The true optimal cumulative cost – You can see RL as learning the perfect heuristic – upon convergence eliminates the complete need for planning)

Reducing width

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Forward pruning

(directly eliminate some of the available actions)

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Forward pruning

(directly eliminate some of the available actions)

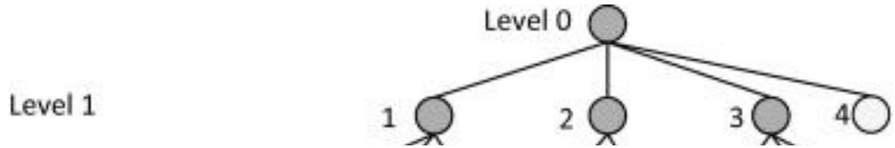
Simplest implementation: **beam search** (only keep best M candidates at every depth)

Beam search

- Expand children
- Select M best children

Beam search

- **Expand children**
- Select M best children



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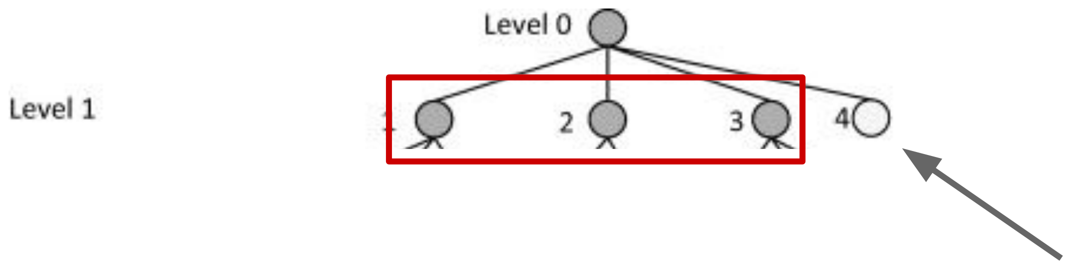

‘forward pruning’

Beam search

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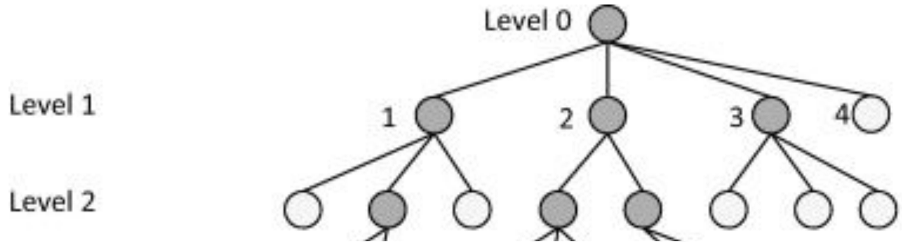

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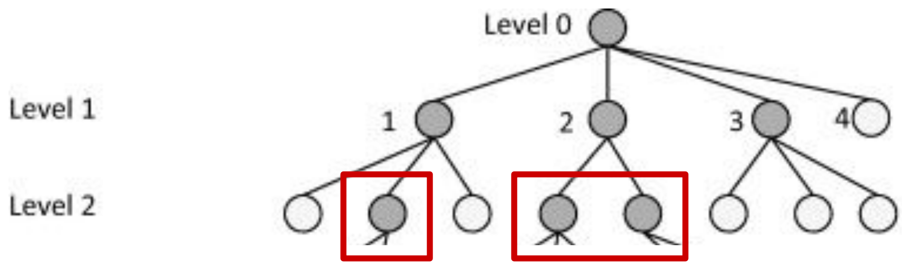

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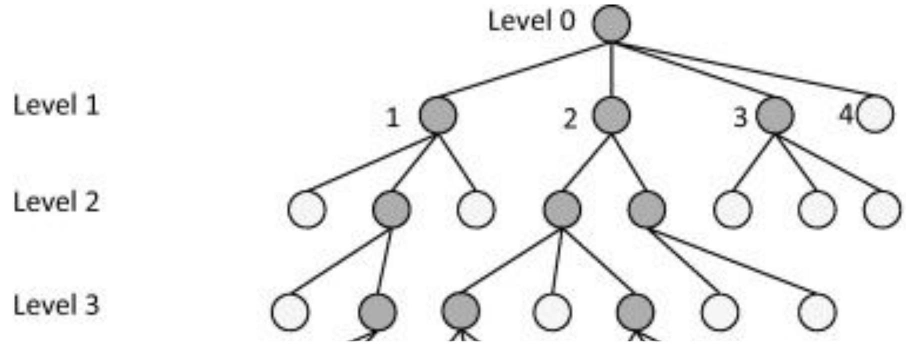

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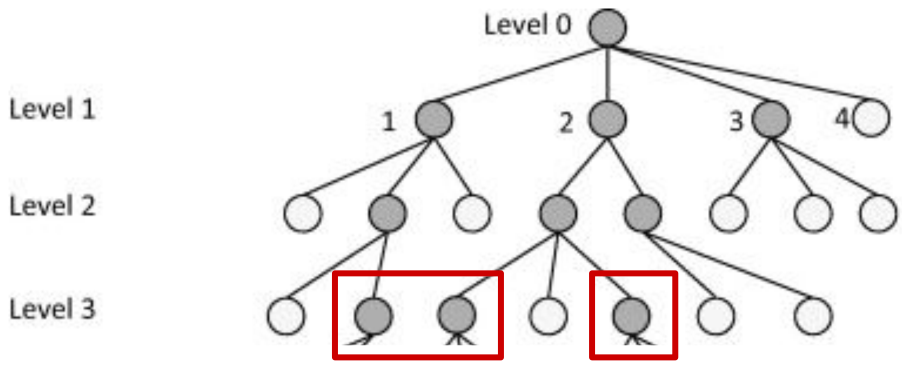

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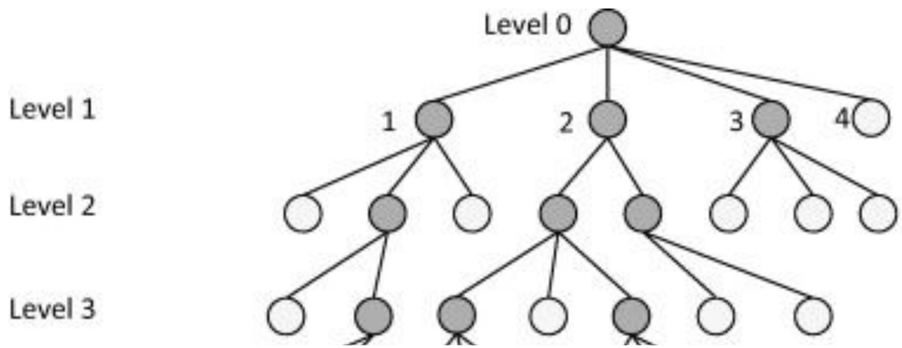

etc.

Beam search

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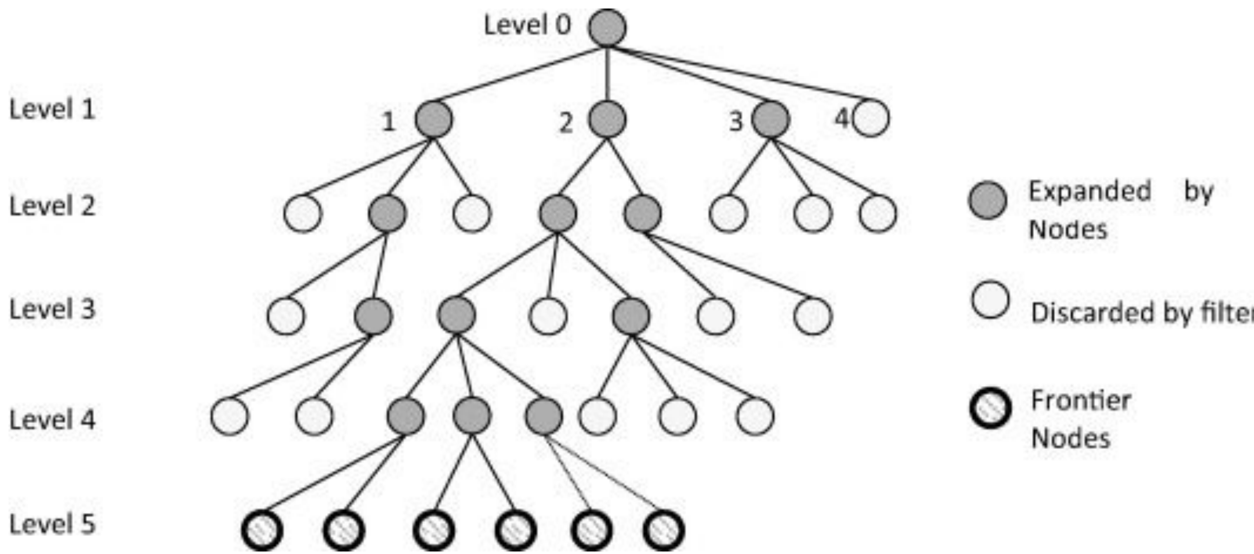

Forward pruning

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Q: What is the risk of forward pruning?

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A: We may prune away optimal actions, and therefore we lose all optimality guarantees

Stochastic dynamics

Classic planning primarily focused on deterministic settings.

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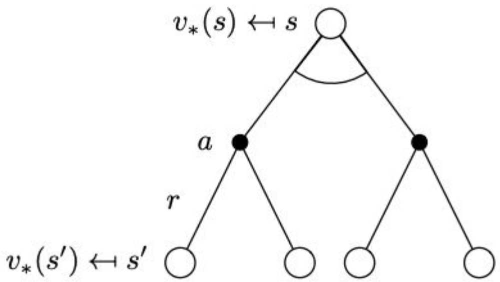
Can we also apply it to the full stochastic MDP setting?



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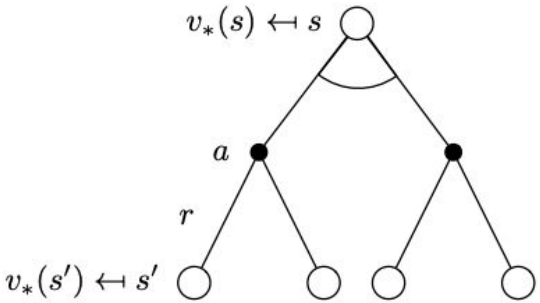
Example: A* —> **AO*** (*AND-OR*)



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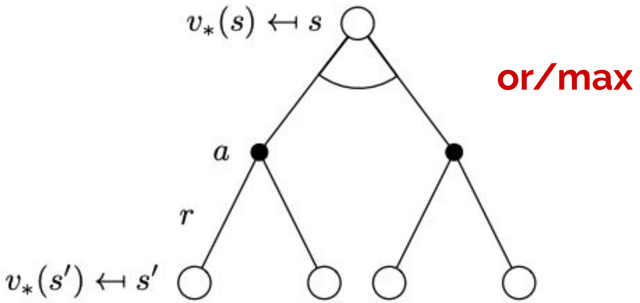

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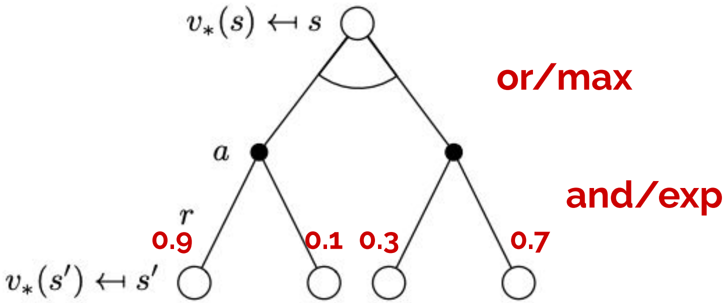

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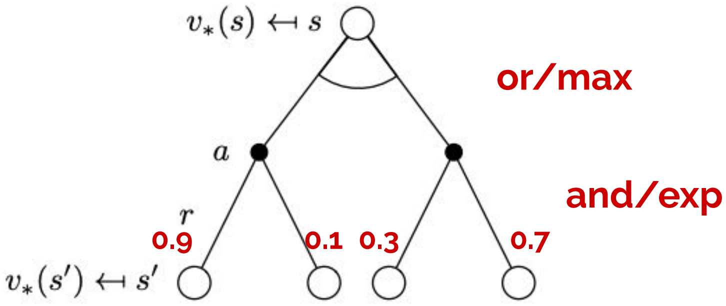

MDP = MAX-EXP graph = AND-OR graph

Stochastic dynamics

Q: What could be the problems of classic search in stochastic settings?

Stochastic dynamics

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A:

1) Need an analytic model (often only a simulator is available, no exact probabilities)

2) Makes the search wide (since we need to expand all possible next states, which gives an extra/double branching factor)

Challenges of heuristic search

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Alternative solution: sample-based planning

Break

3. Sample-based Planning

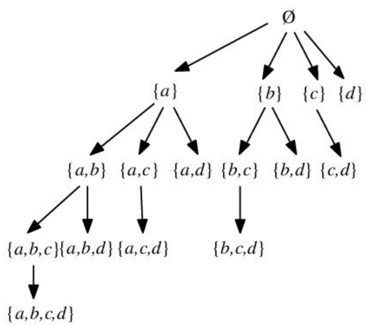
Sample-based planning

Sample-based planning

'Roll-out algorithms'

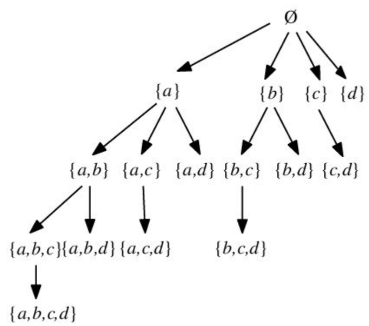
(Sutton & Barto)

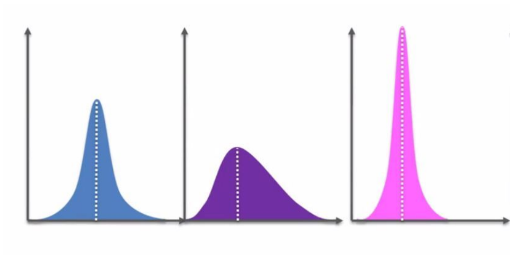
Sample-based planning



Replace the concept of systematic enumeration

Sample-based planning





Replace the concept of systematic enumeration

With statistical/probabilistic/Monte Carlo estimation of action values

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Depth:

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<u>Some algorithms still retain probabilistic convergence guarantees (in the limit)</u>

Monte Carlo Search

Monte Carlo Search

Main idea:

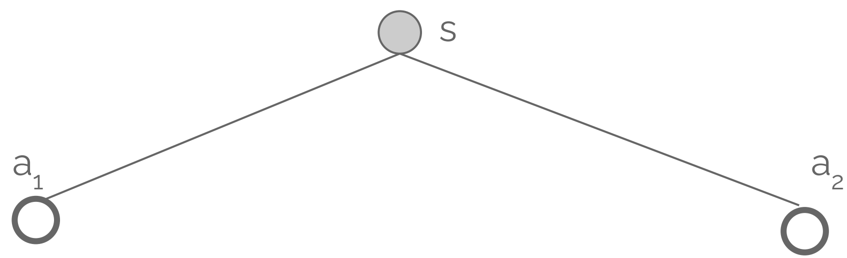
1) Use roll-outs to estimate of mean return of each action (‘*Monte Carlo estimation*’)

2) Select the action with the highest mean return (‘*Uniform bandit algorithm*’)

Monte Carlo Search

Monte Carlo Search

For each of
the A actions

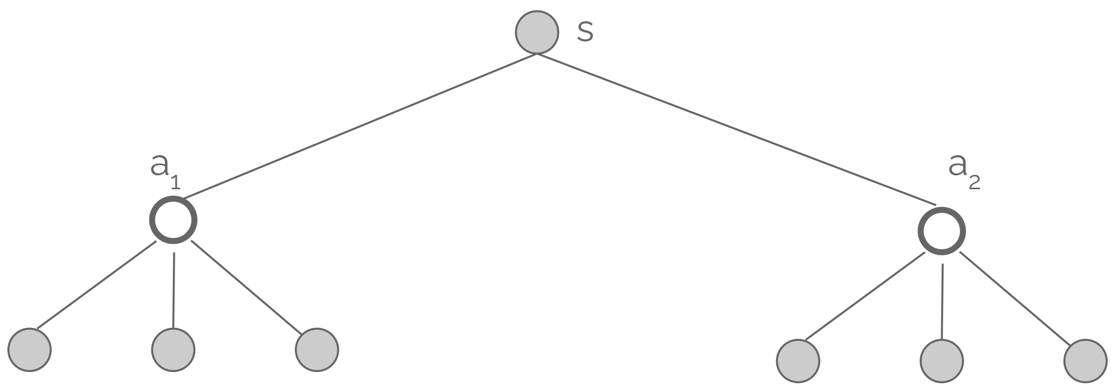


A=2

Monte Carlo Search

For each of
the A actions

Sample N
trajectories



A=2

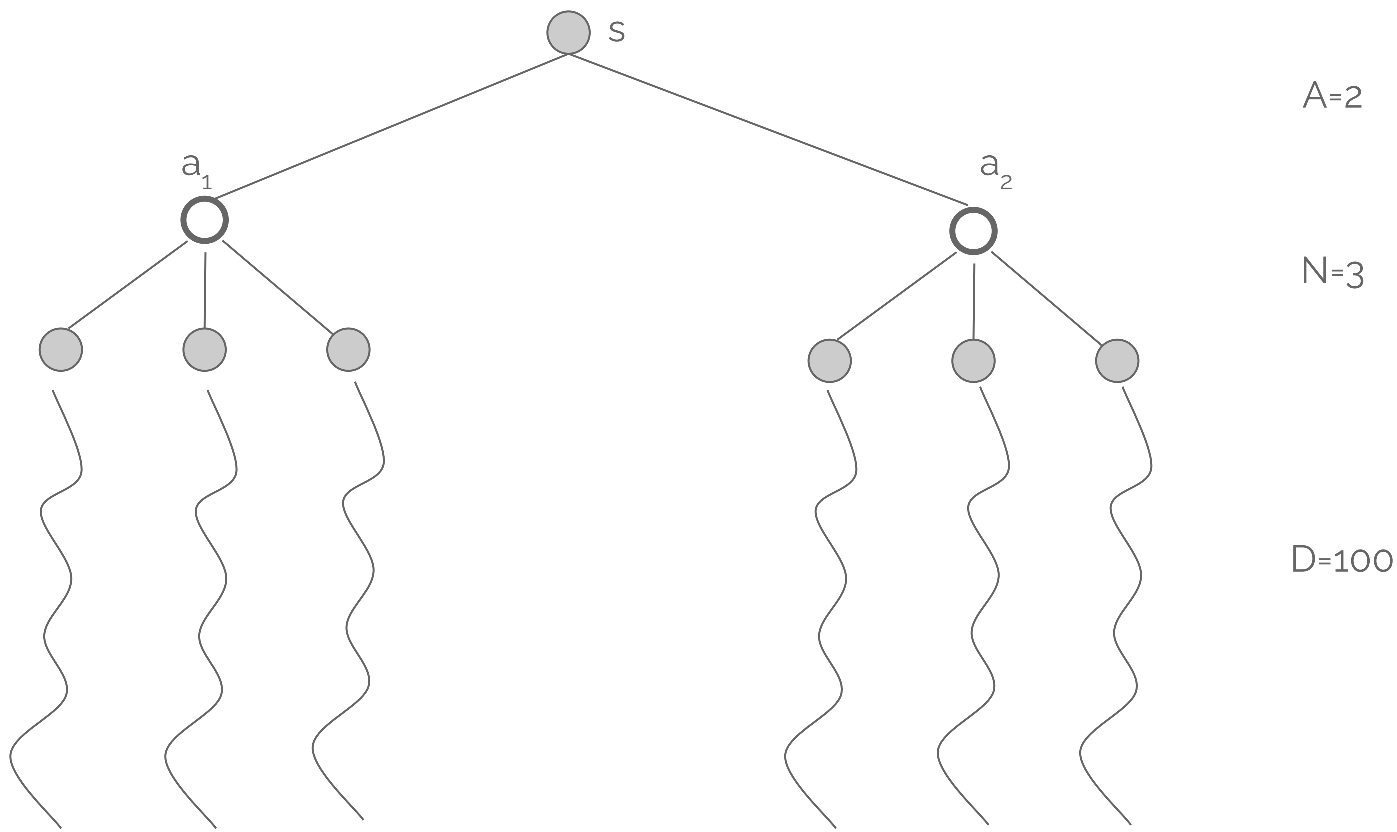
N=3

Monte Carlo Search

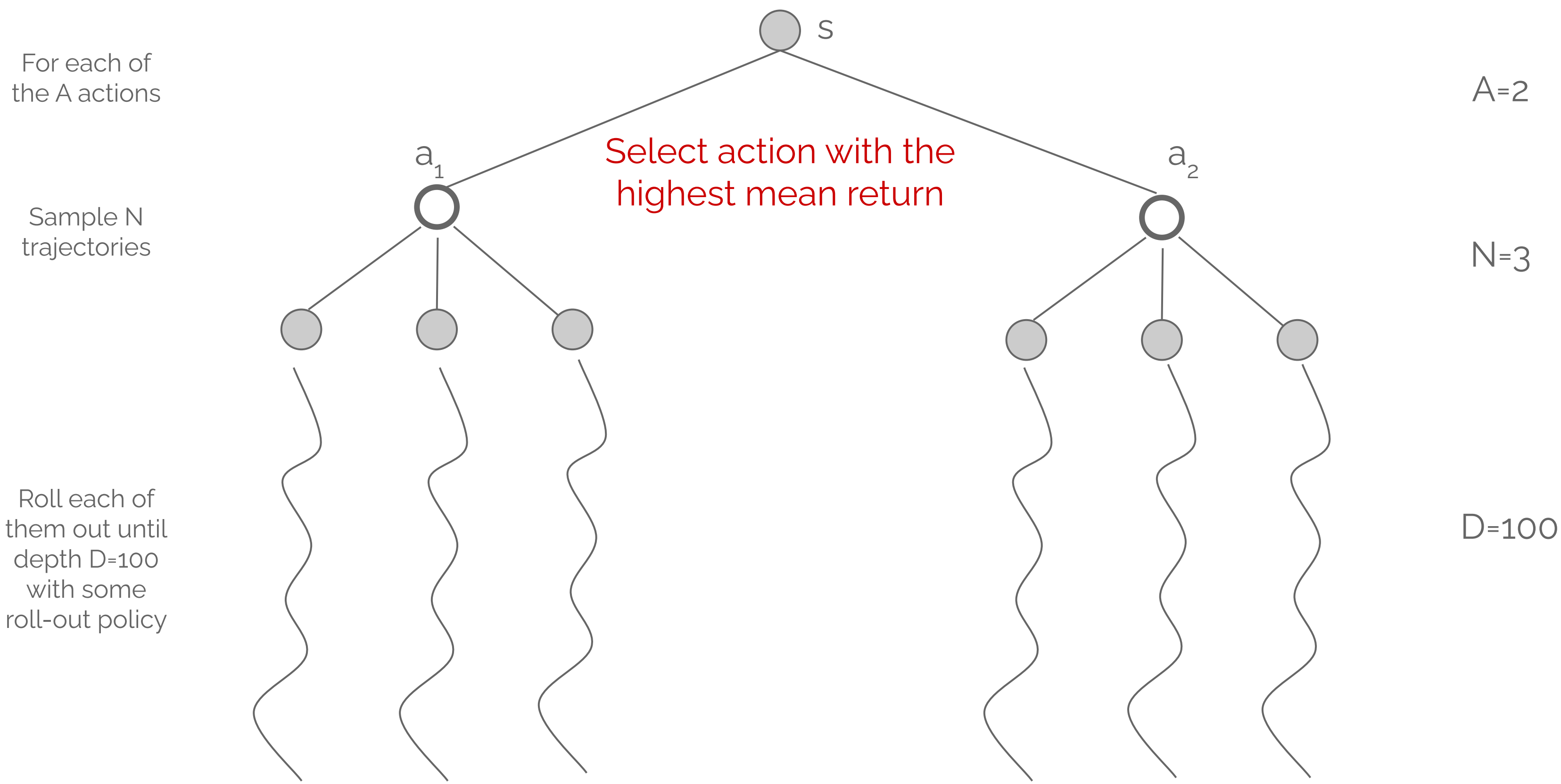
For each of the A actions

Sample N trajectories

Roll each of them out until depth D=100 with some roll-out policy



Monte Carlo Search



Monte Carlo Search

<u>Sample complexity</u>: ?

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Sample N trajectories

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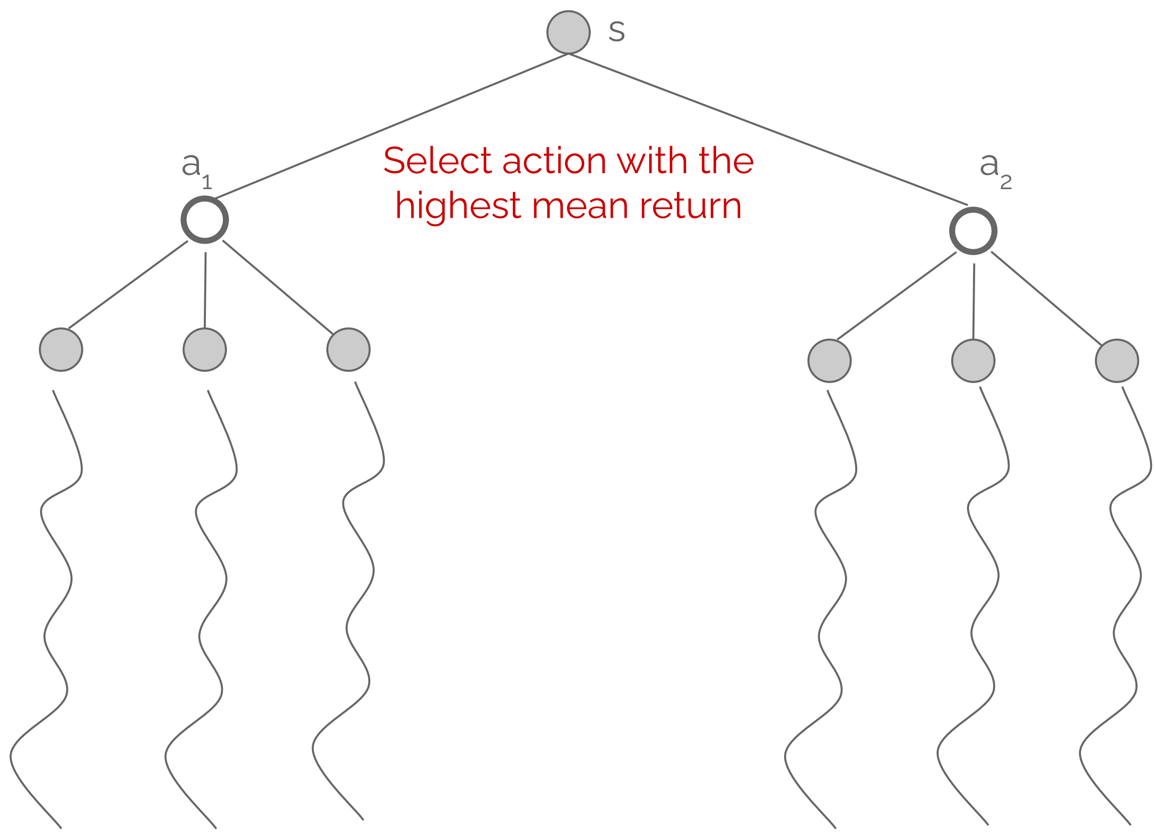

A=2

N=3

D=100

Monte Carlo Search

Sample complexity: A · N · D

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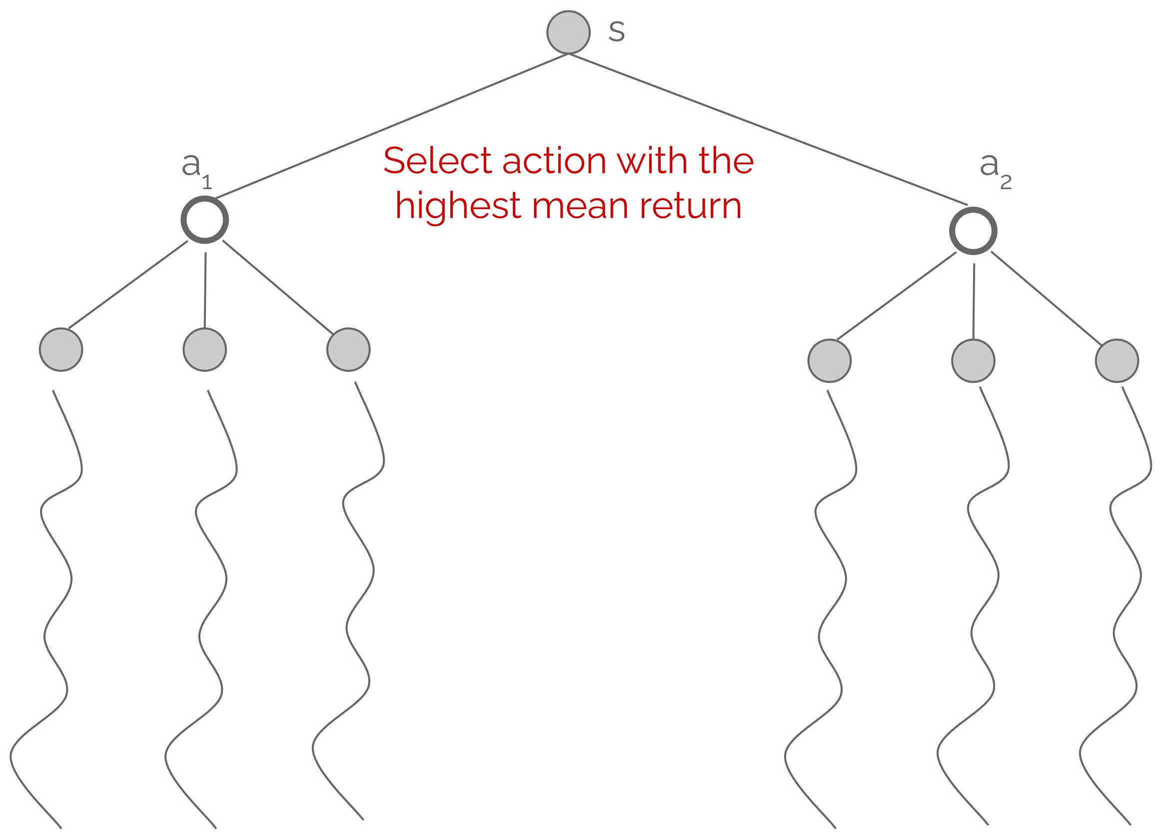

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N=3

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Monte Carlo Search: Roll-out policy

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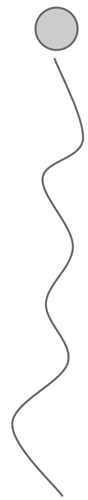
Performance of Monte Carlo search depends on the quality of the roll-out policy



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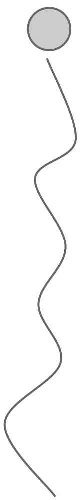
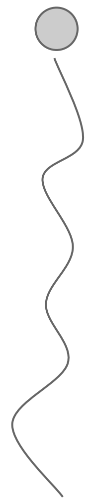
- Uninformed version: Random policy (default choice)



Monte Carlo Search: Roll-out policy

Performance of Monte Carlo search depends on the quality of the roll-out policy

- Uninformed version: Random policy (default choice)
- Informed version: May use better prior roll-out policy when available



Monte Carlo Search: Interpretation

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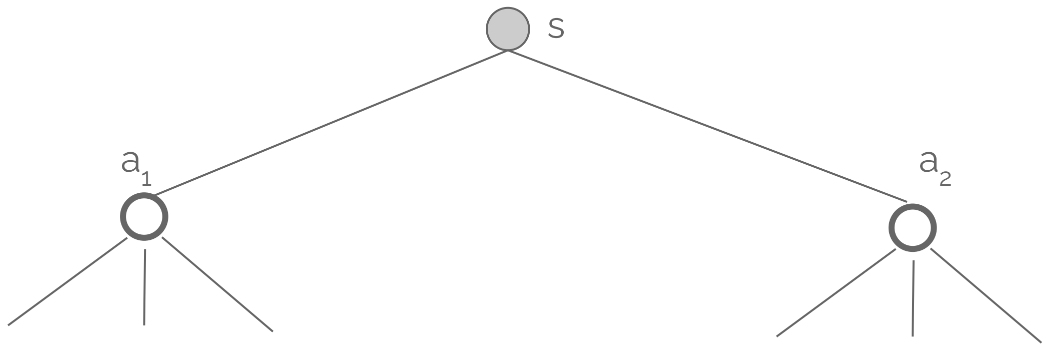
What does the mean return of each action in Monte Carlo Search actually estimate?



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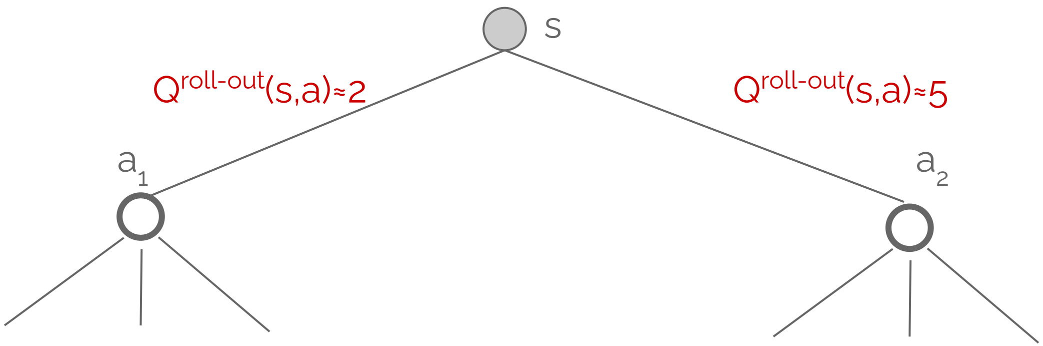

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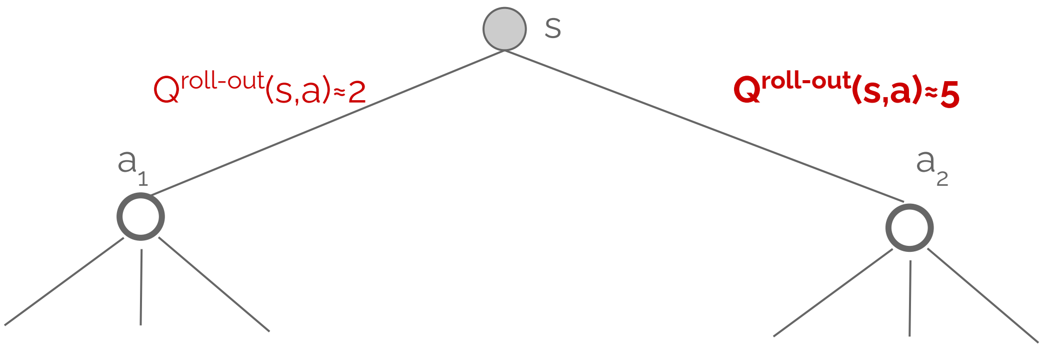

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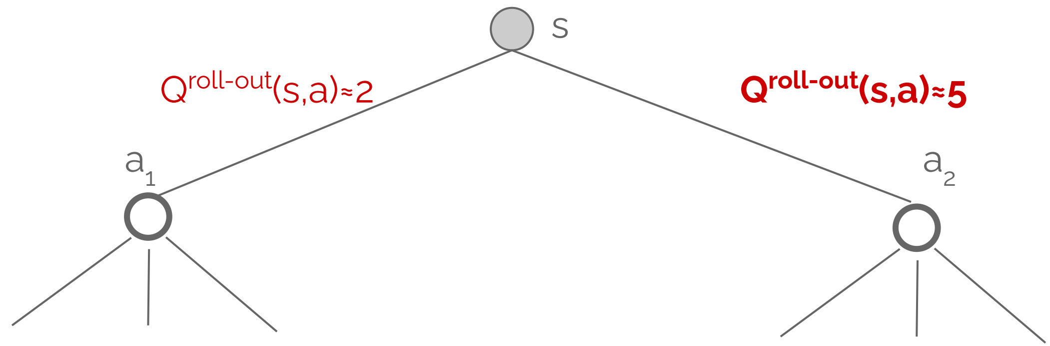

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- A form of local, one-step policy improvement over the prior roll-out policy

Monte Carlo Search: Downside

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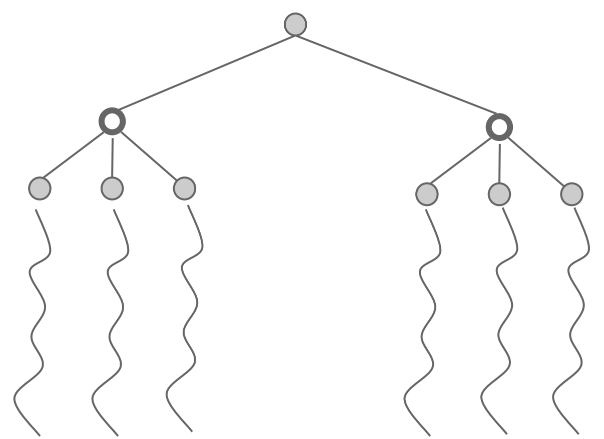

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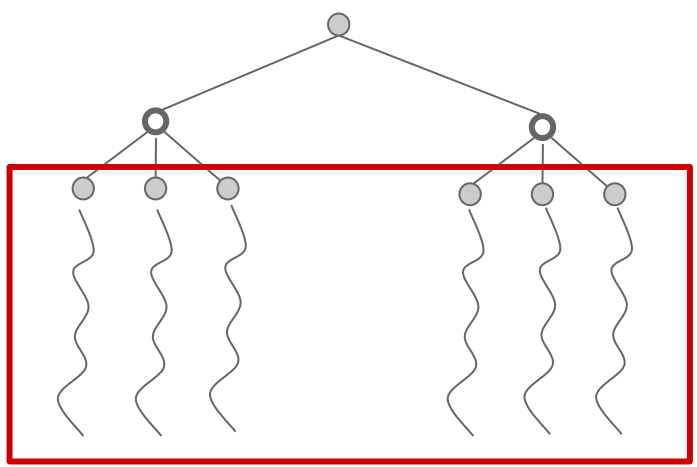


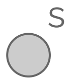
black box: not changing your policy based on the search

Sparse sampling

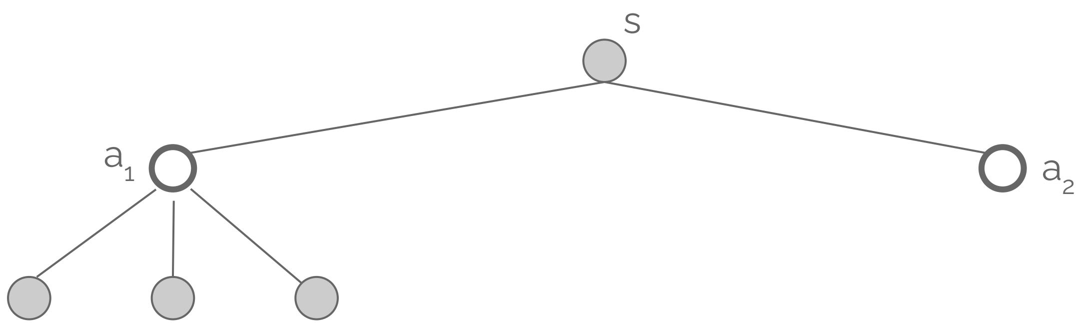
Sparse sampling

Main idea: Repeat the same process at deeper levels

Sparse sampling



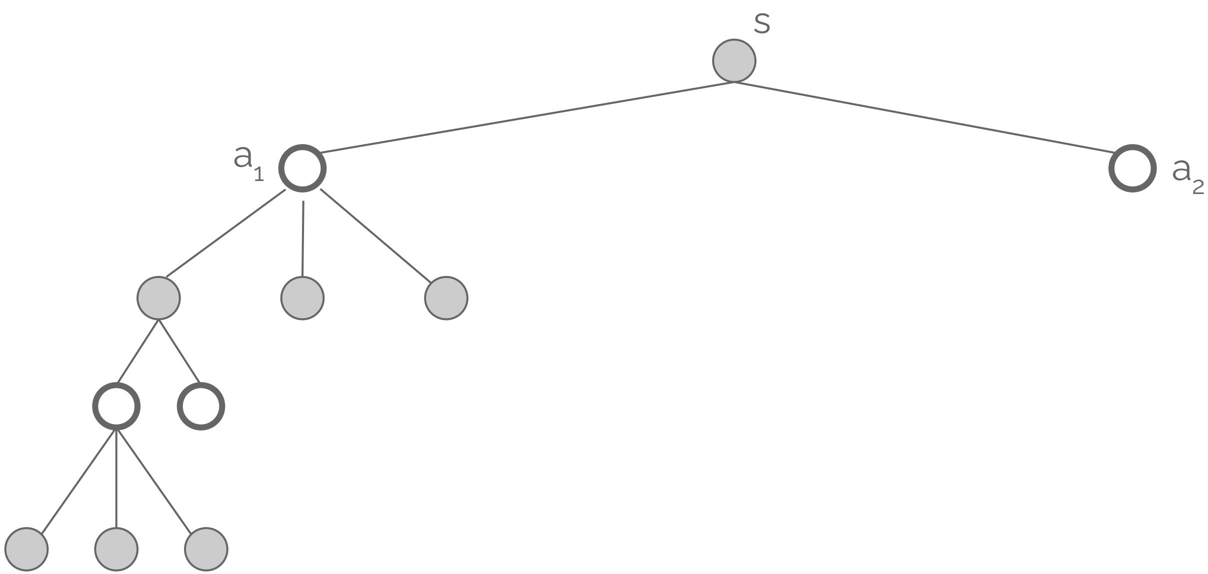
Sparse sampling



A=2

N=3

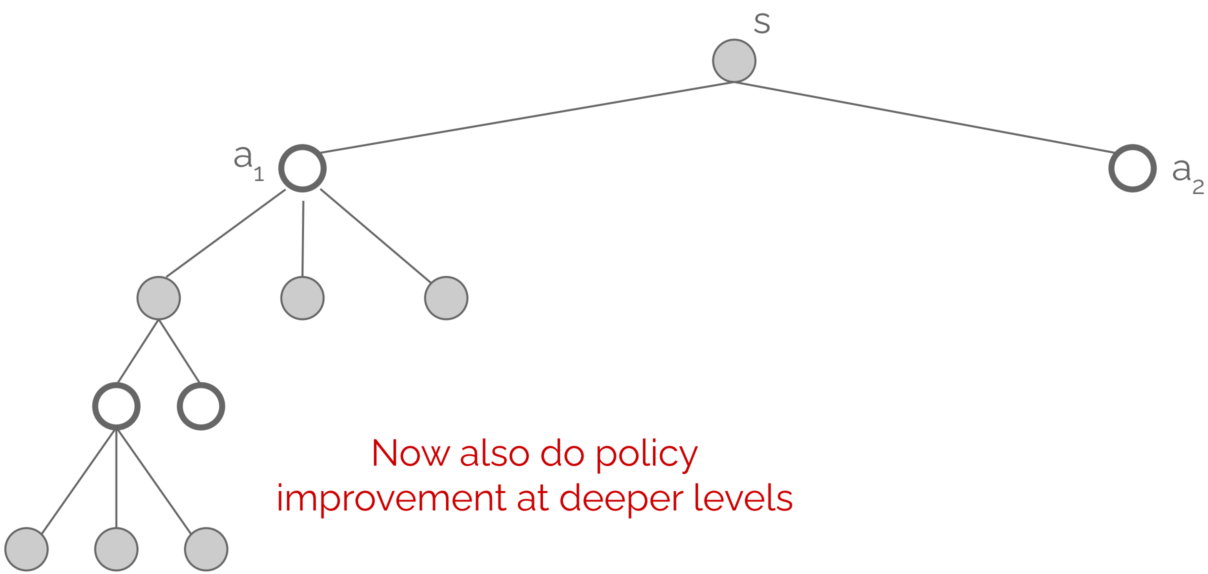
Sparse sampling



A=2

N=3

Sparse sampling

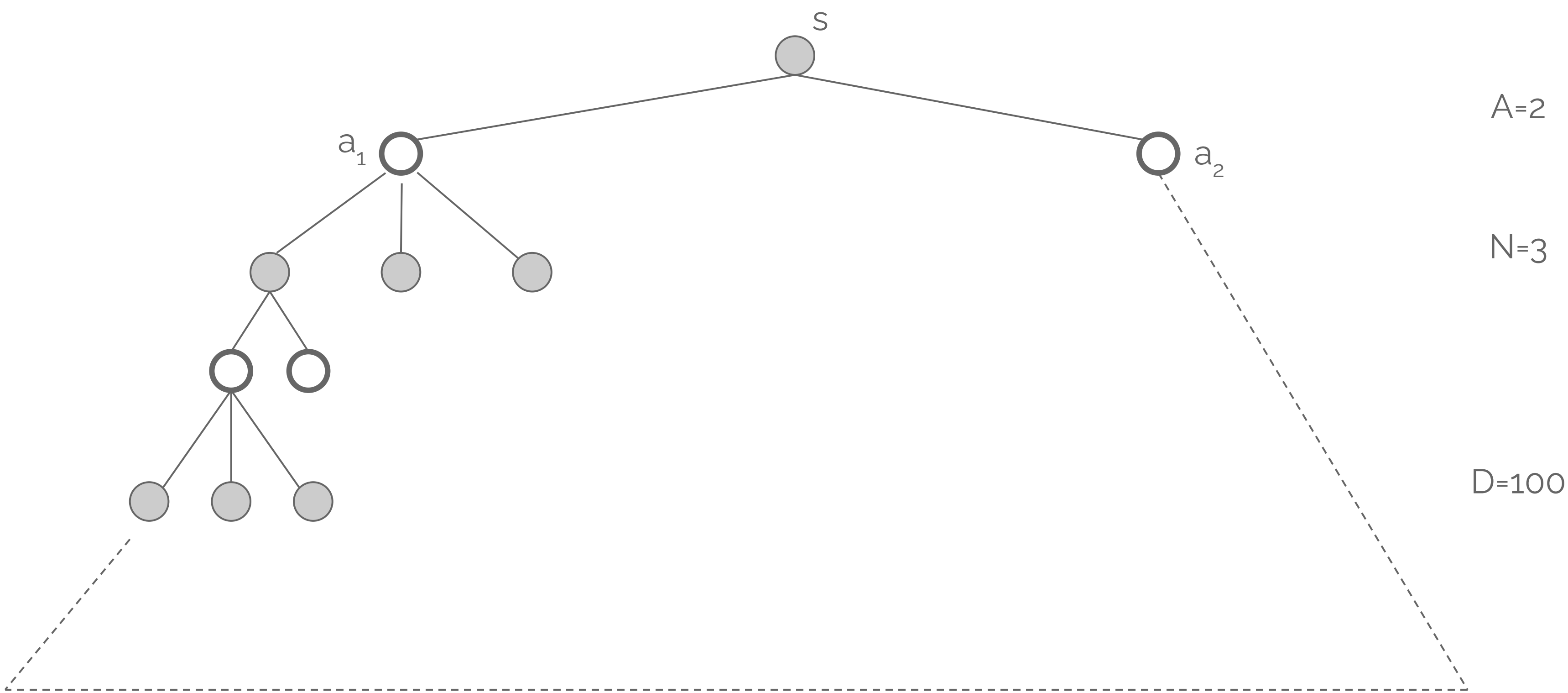


A=2

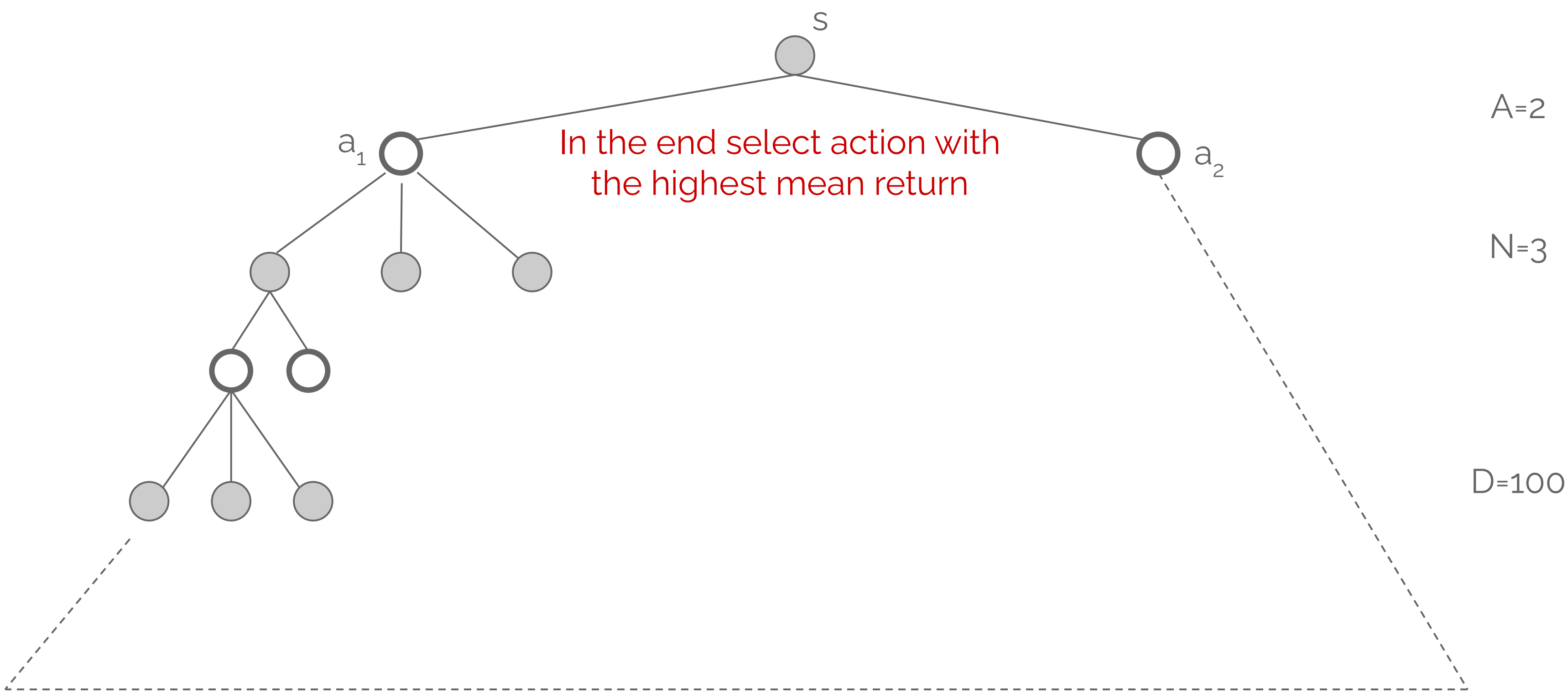
N=3

Now also do policy
improvement at deeper levels

Sparse sampling

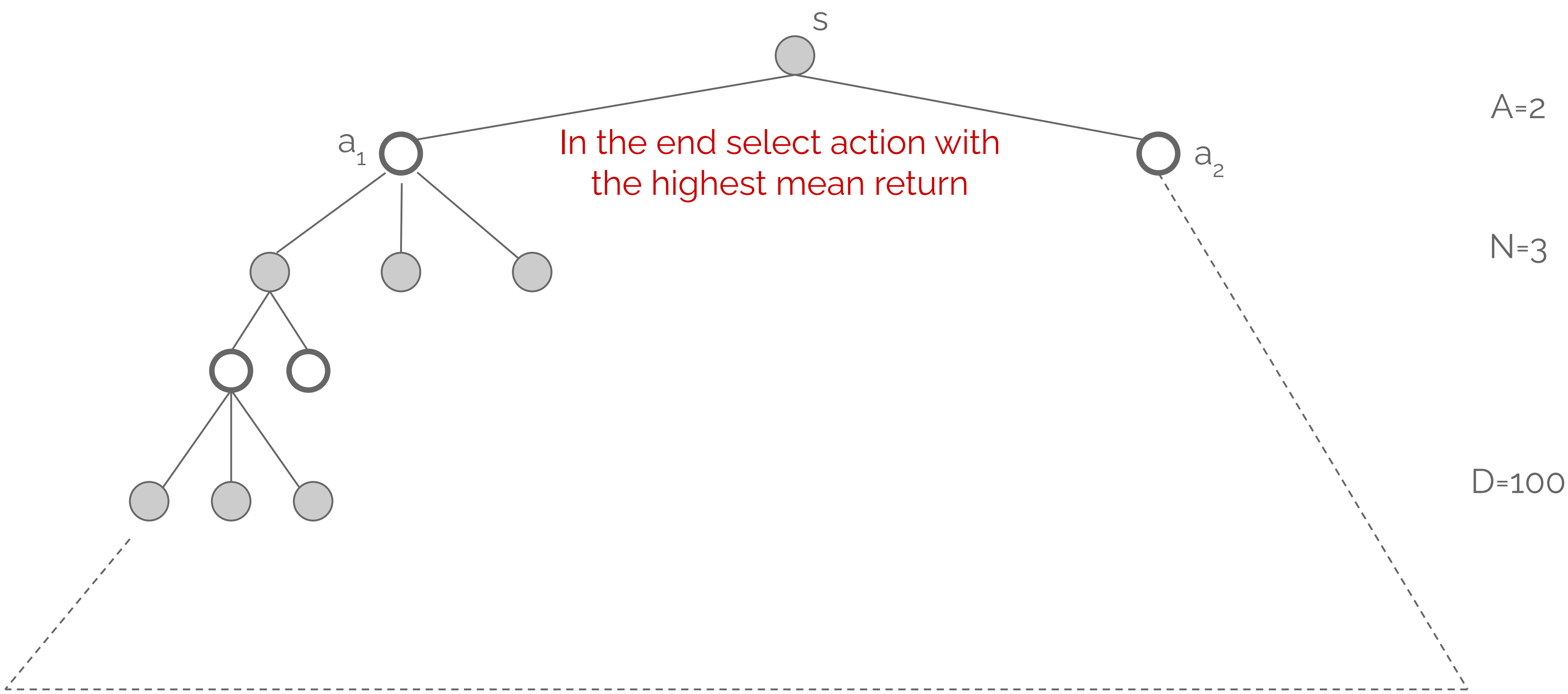


Sparse sampling



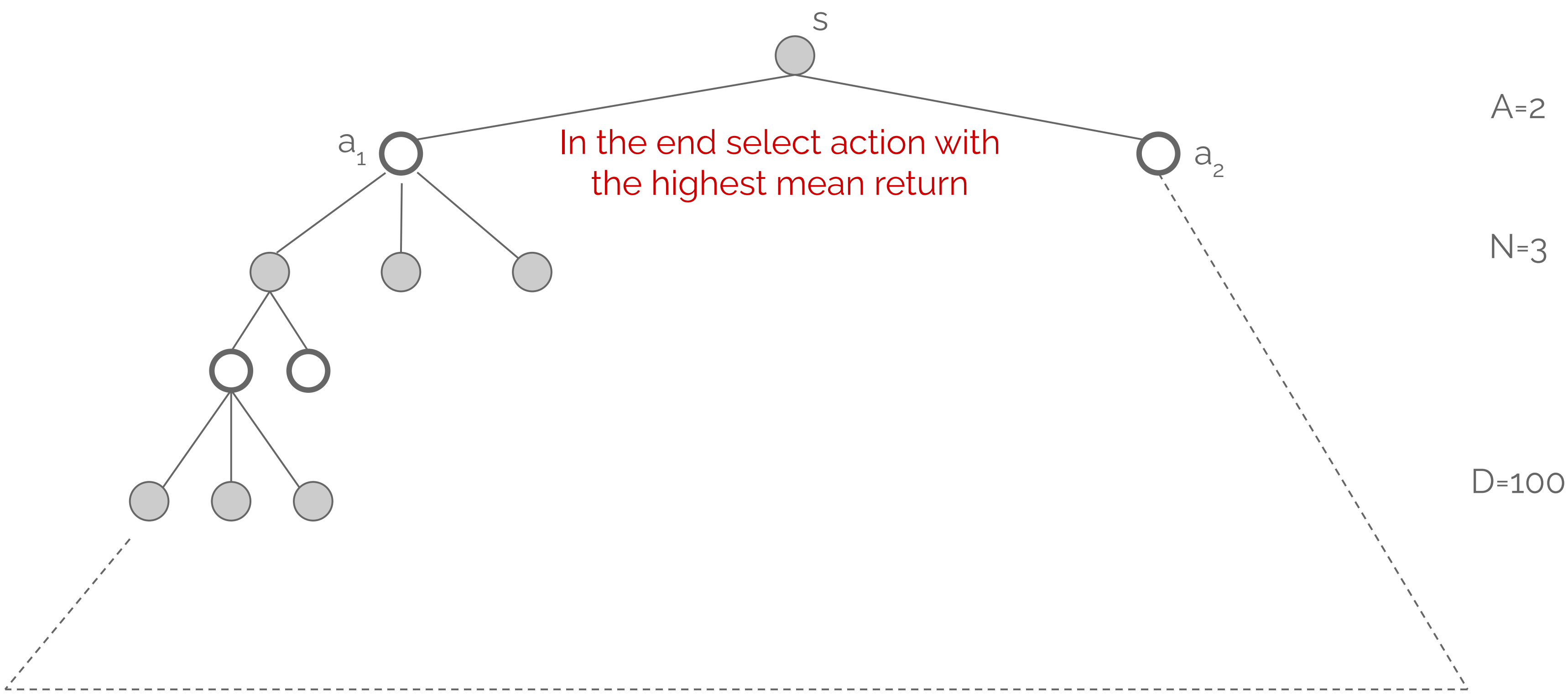
Sample complexity: ?

Sparse sampling



Sample complexity: $(A \cdot N)^D$

Sparse sampling



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Can you think of a solution?

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Can you think of a solution?

- Adaptive Monte Carlo methods → replace Uniform sampling with an adaptive bandit algorithm (Ch. 2) that focuses in directions where initial samples perform well (trading-off exploration & exploitation).

Monte Carlo Tree Search (MCTS)

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Main idea: iteratively apply adaptive bandit algorithm at every depth

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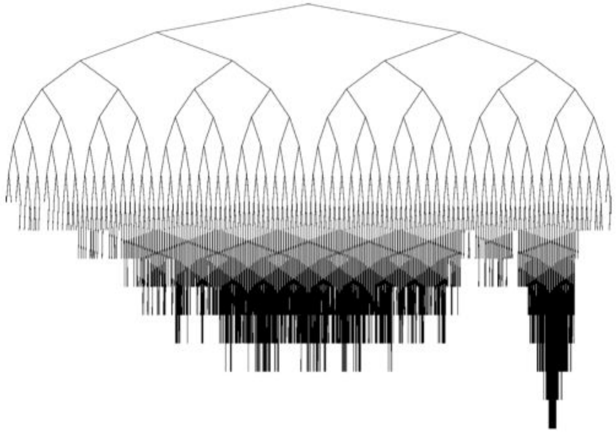
Naturally produces an asymmetric search tree



Monte Carlo Tree Search (MCTS)

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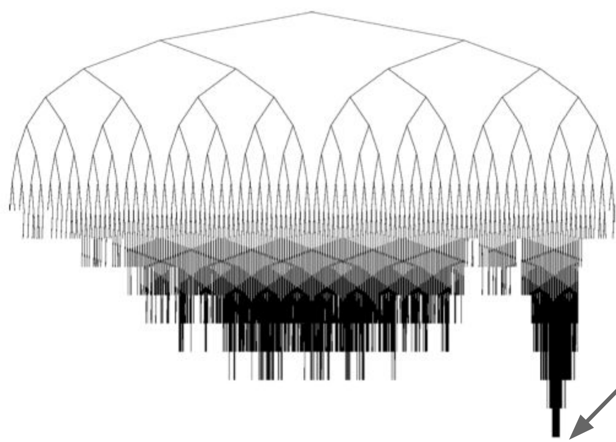


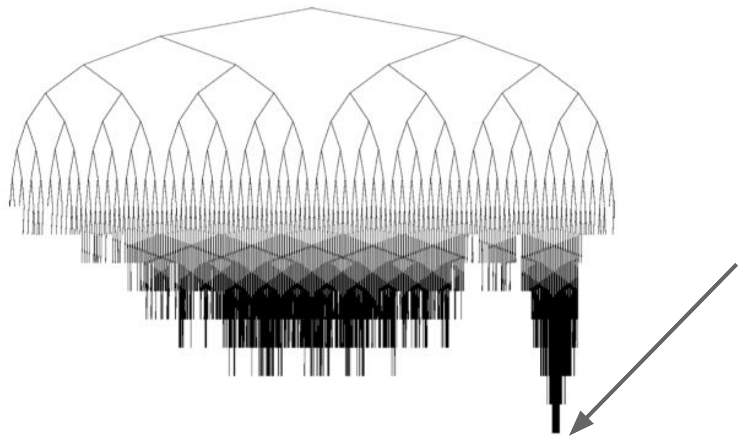
Extends deeper in directions where initial samples giving promising returns

(sample-based equivalent of prioritized search)

Monte Carlo Tree Search (MCTS)

Main idea: iteratively apply adaptive bandit algorithm at every depth

Naturally produces an asymmetric search tree



Extends deeper in directions where initial samples giving promising returns

(sample-based equivalent of prioritized search)

Breakthrough performance in the game of Go

Four phases of MCTS

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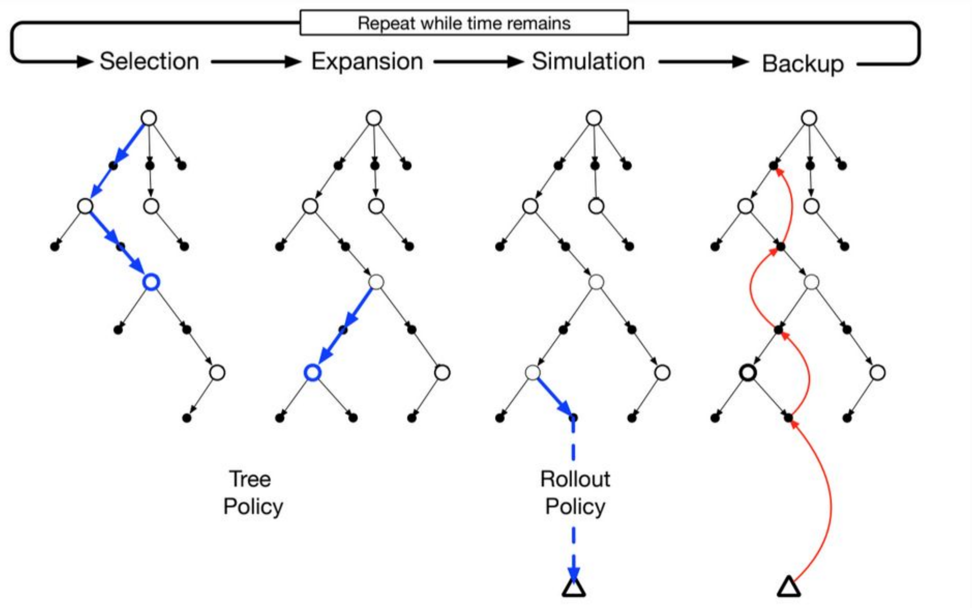
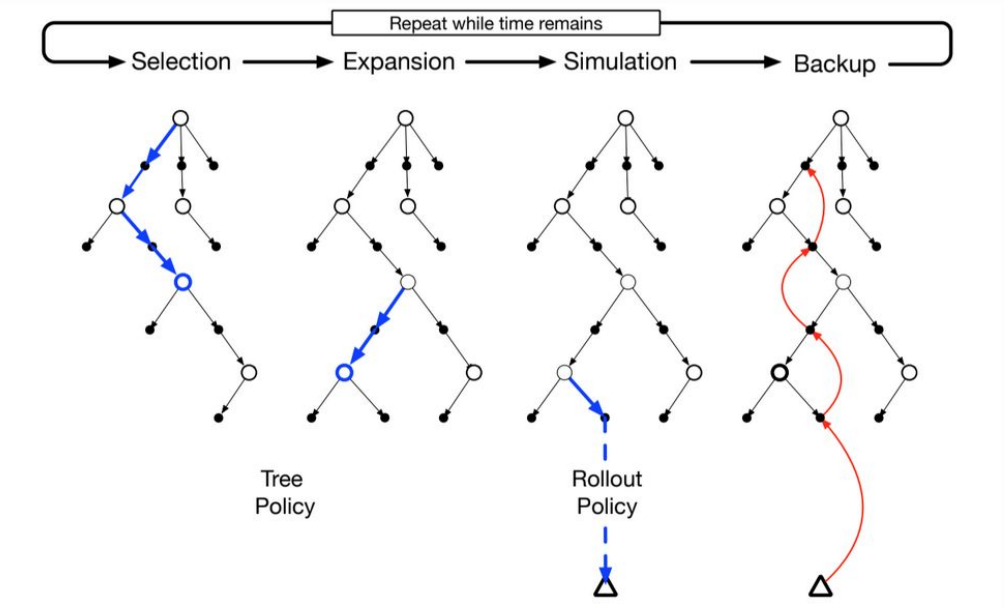


Four phases of MCTS

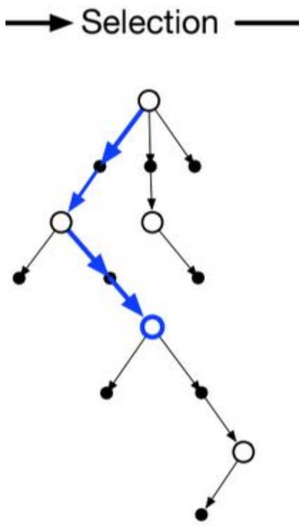


Each iteration makes one roll-out, that moves through four phases.

Selection

Selection

- Apply a bandit algorithm to select the most promising action (balances exploration & exploitation)

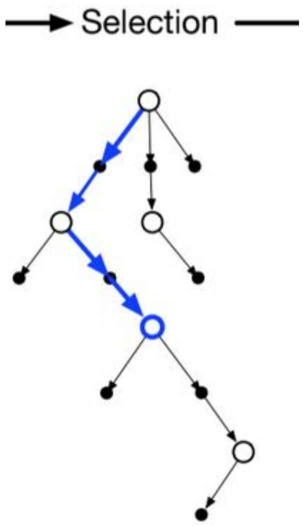


Selection

- Apply a bandit algorithm to select the most promising action (balances exploration & exploitation)

- Most common choice:
  Upper Confidence Bounds applied to Trees (**UCT**)

$$\pi_{UCT}(s) = \arg\max_a Q(s,a) + c\sqrt{\frac{\ln n(s)}{n(s,a)}}$$



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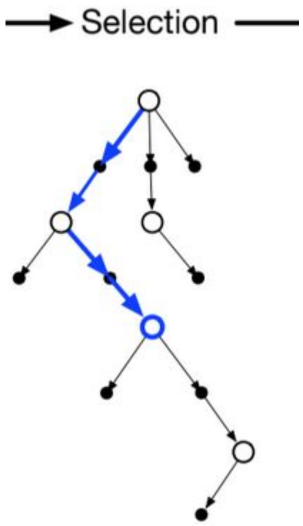

$$\pi_{UCT}(s) = \arg\max_a Q(s,a) + c\sqrt{\frac{\ln n(s)}{n(s,a)}}$$

Select the action with the highest

Selection

- Apply a bandit algorithm to select the most promising action (balances exploration & exploitation)

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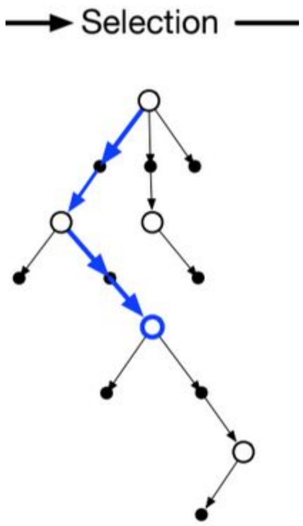
$$\pi_{UCT}(s) = \arg\max_a Q(s,a) + c\sqrt{\frac{\ln n(s)}{n(s,a)}}$$

mean return of previous traces
(exploitation)



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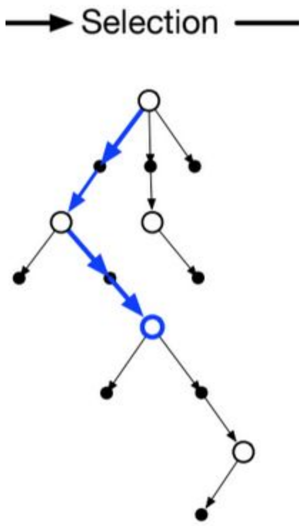

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c = constant we empirically tune
(higher c → more exploration)

Selection

- Apply a bandit algorithm to select the most promising action (balances exploration & exploitation)

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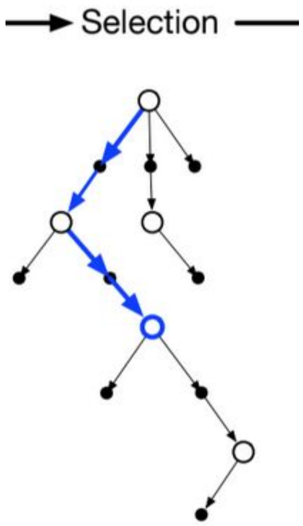

$$\pi_{UCT}(s) = \arg\max_a Q(s,a) + c\sqrt{\frac{\ln n(s)}{n(s,a)}}$$

n(s) = number of traces through state
n(s,a) = number of traces through state-action
(exploration: lower n(s,a) → higher second term)

Selection

- Apply a bandit algorithm to select the most promising action (balances exploration & exploitation)

- Most common choice:
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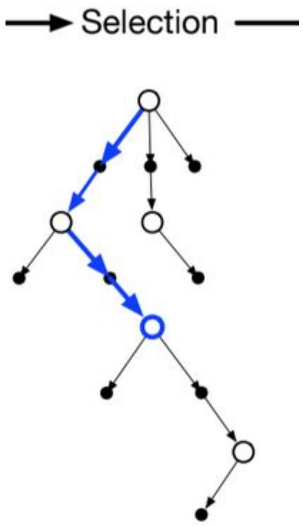
$$\pi_{UCT}(s) = \arg\max_a Q(s,a) + c\sqrt{\frac{\ln n(s)}{n(s,a)}}$$

- What is the UCT value of an untried action [n(s,a)=0]?



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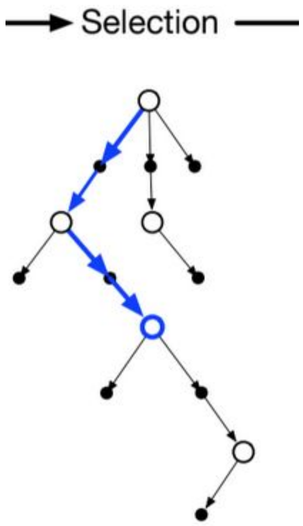


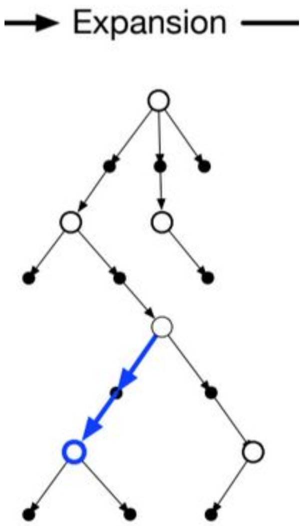
$$\pi_{UCT}(s) = \arg\max_a Q(s,a) + c\sqrt{\frac{\ln n(s)}{n(s,a)}}$$

- What is the UCT value of an untried action [n(s,a)=0]?
  - We treat the second term as infinity (divide over 0) and therefore **always select an untried action when available (=expand)**

Expansion

Expansion

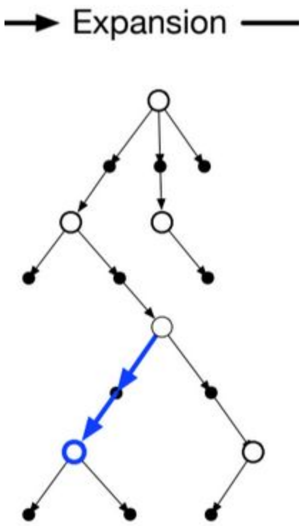
- Once we reach an unvisited action, expand it (i.e. add child state and its actions to the tree)



Expansion

- Once we reach an unvisited action, expand it (i.e. add child state and its actions to the tree)

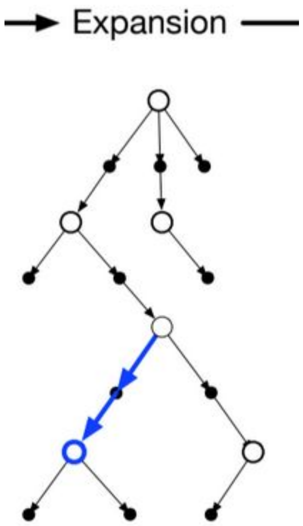
- Each iteration expands the tree with only one new state. Why?



Expansion

- Once we reach an unvisited action, expand it (i.e. add child state and its actions to the tree)

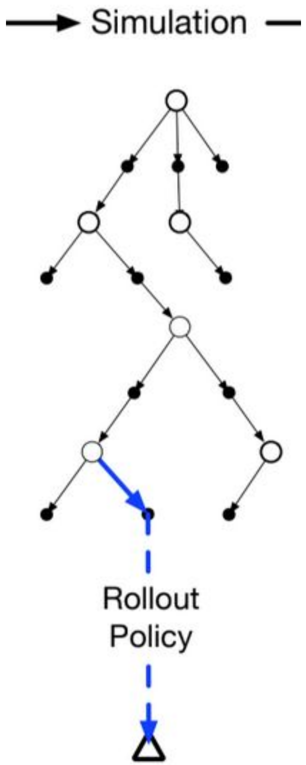
- Each iteration expands the tree with only one new state. Why?
    - Could store everything below but eats away memory and compute. We only start storing a deeper state once we repeatedly visited that direction.



Simulation

Simulation

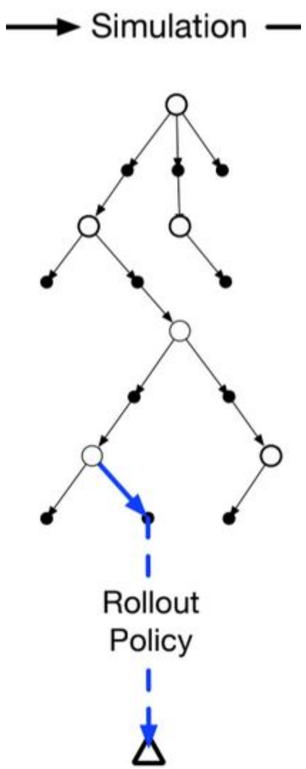
Again use Monte Carlo roll-out as an estimate of the value of the expanded state



Simulation

Again use Monte Carlo roll-out as an estimate of the value of the expanded state

- Default = random policy, can use better prior policy when available.

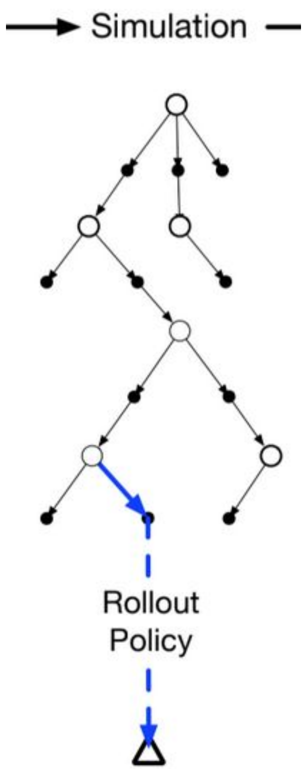


Simulation

Again use Monte Carlo roll-out as an estimate of the value of the expanded state

- Default = random policy, can use better prior policy when available.

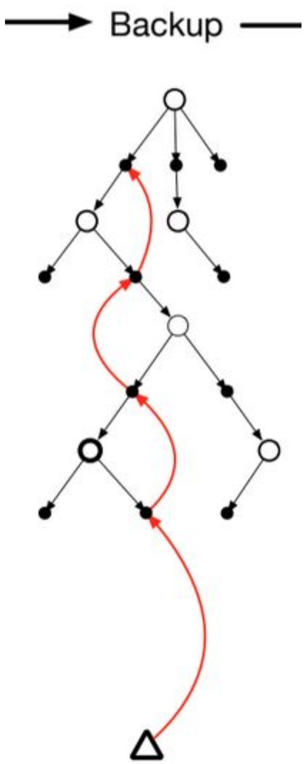
- Note max total depth D (so if current leaf at depth 5 with D=100, then you roll-out for length 95)



Backup

Backup

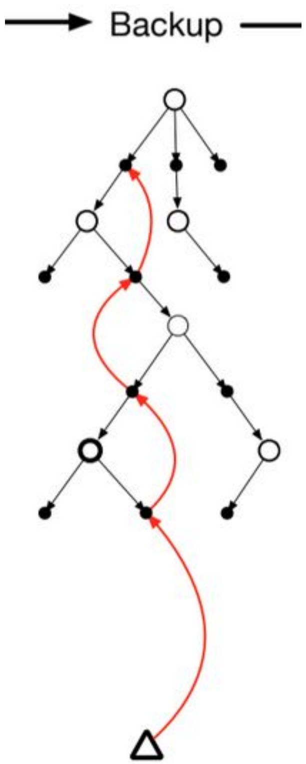
Update the statistics throughout the tree to direct the next iteration



Backup

Update the statistics throughout the tree to direct the next iteration

- Action nodes: store visit count **n(s,a)** and mean return **Q(s,a)**

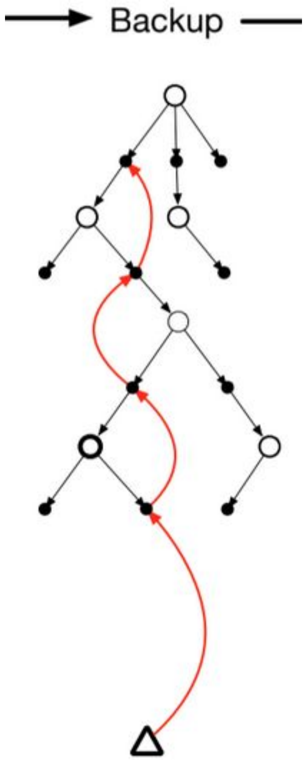


Backup

Update the statistics throughout the tree to direct the next iteration

- Action nodes: store visit count **n(s,a)** and mean return **Q(s,a)**

May also store the return sum of all traces through (s,a) as **R_{sum}(s,a)** and compute **Q(s,a) = R_{sum}(s,a)/n(s,a)**



Backup

Update the statistics throughout the tree to direct the next iteration

- Action nodes: store visit count **n(s,a)** and mean return **Q(s,a)**
- State nodes: store visit count **n(s)**

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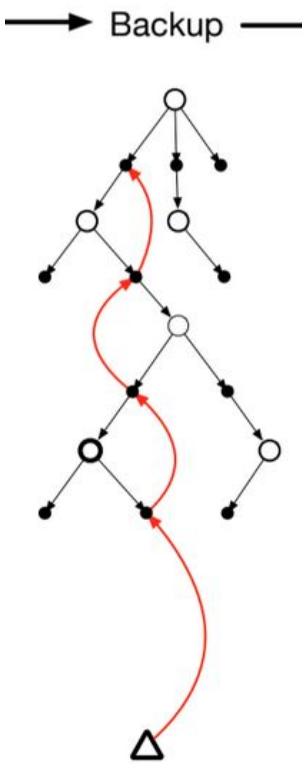


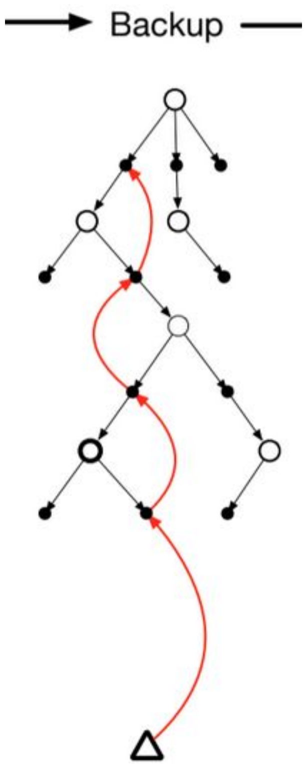
Backup

Update the statistics throughout the tree to direct the next iteration

- Action nodes: store visit count **n(s,a)** and mean return **Q(s,a)**
- State nodes: store visit count **n(s)**

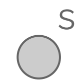
Or compute them as $\boldsymbol{\Sigma_a}$ **n(s,a)** once needed

May also store the return sum of all traces through (s,a) as $\mathbf{R_{sum}(s,a)}$ and compute $\mathbf{Q(s,a) = R_{sum}(s,a)/n(s,a)}$



Monte Carlo Tree Search

1. Select
2. Expand
3. Roll-out
4. Back-up



Monte Carlo Tree Search

1. **Select**
2. Expand
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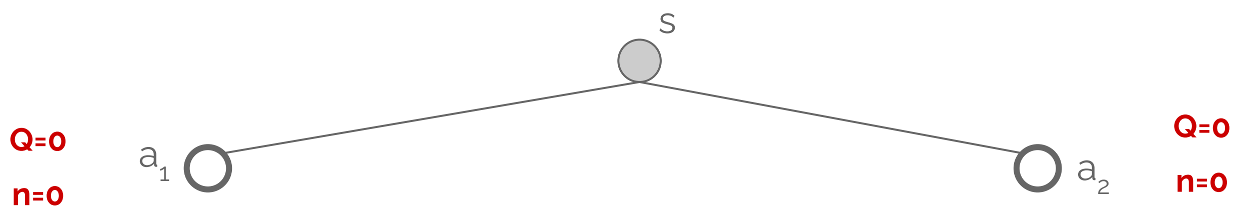

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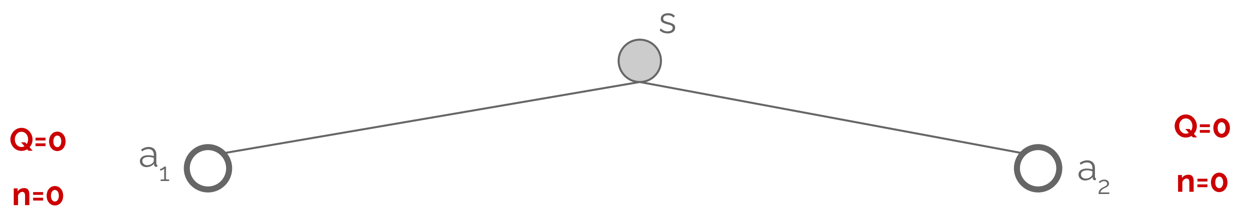
Monte Carlo Tree Search



Initialize mean action return (**Q(s,a)**) and count (**n(s,a)**) to 0

Monte Carlo Tree Search

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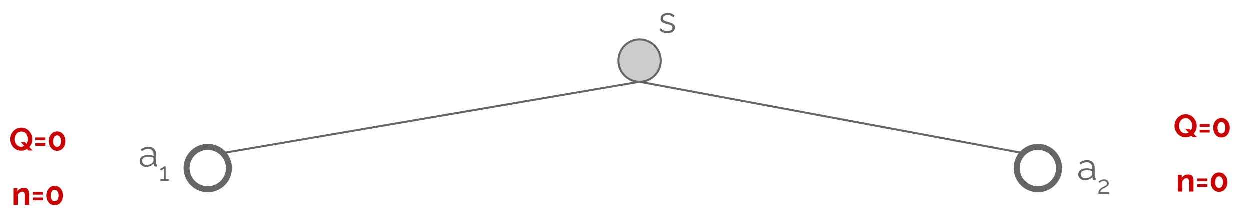


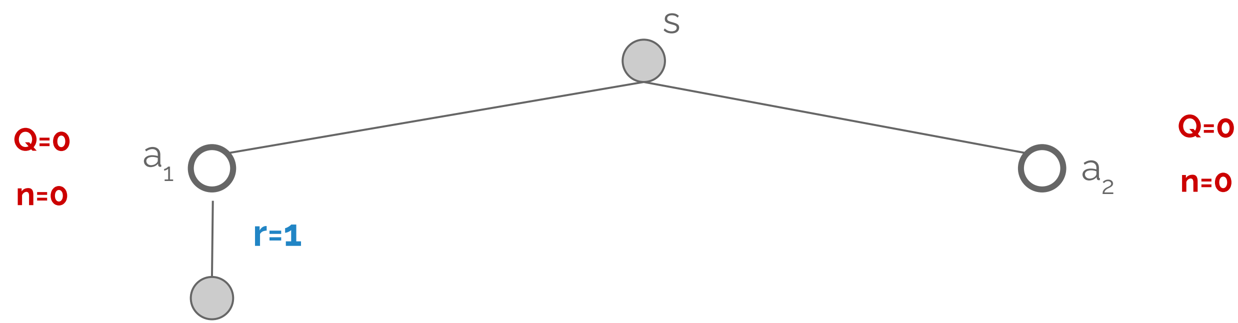
Select next action based on UCT rule:

$$\pi_{UCT}(s) = \arg\max_a Q(s,a) + c\sqrt{\frac{\ln n(s)}{n(s,a)}}$$

Both actions untried (n=0), randomly pick one

Monte Carlo Tree Search

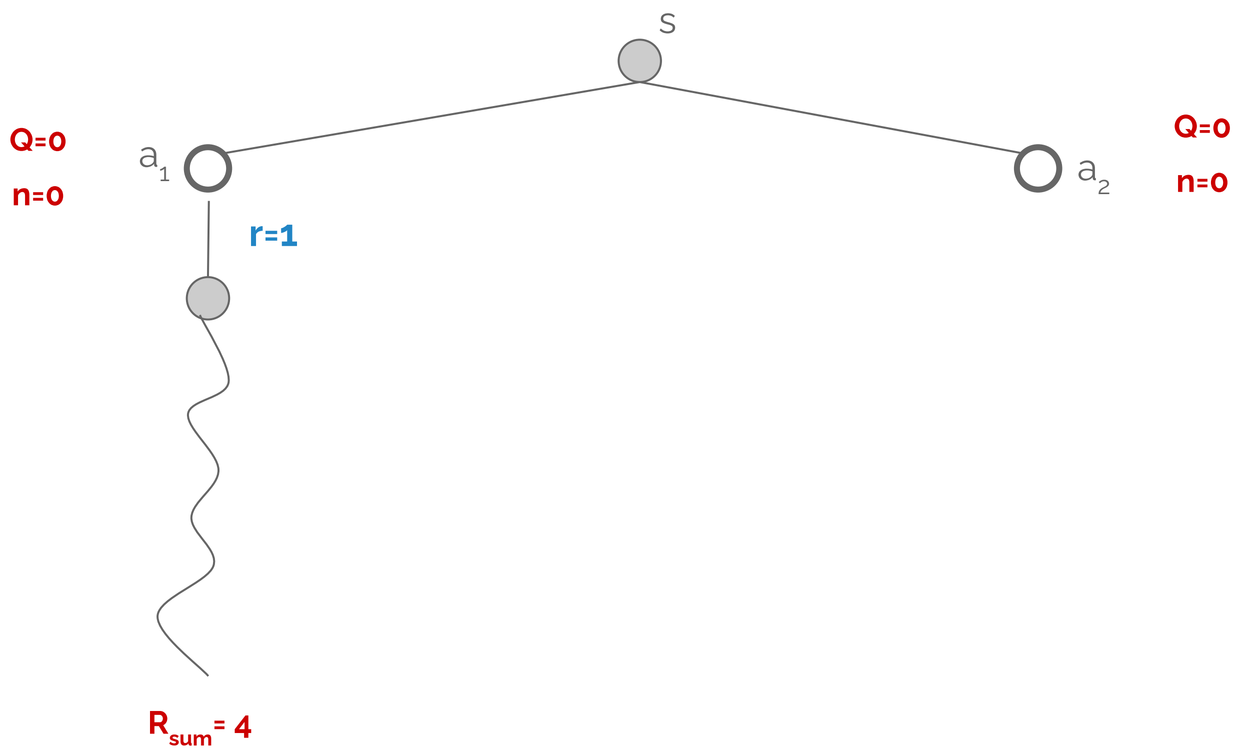
1. Select
2. **Expand**
3. Roll-out
4. Back-up



We expand the tree once we encounter an untried action

Monte Carlo Tree Search

1. Select
2. Expand
3. **Roll-out**
4. Back-up



Monte Carlo Tree Search

1. Select
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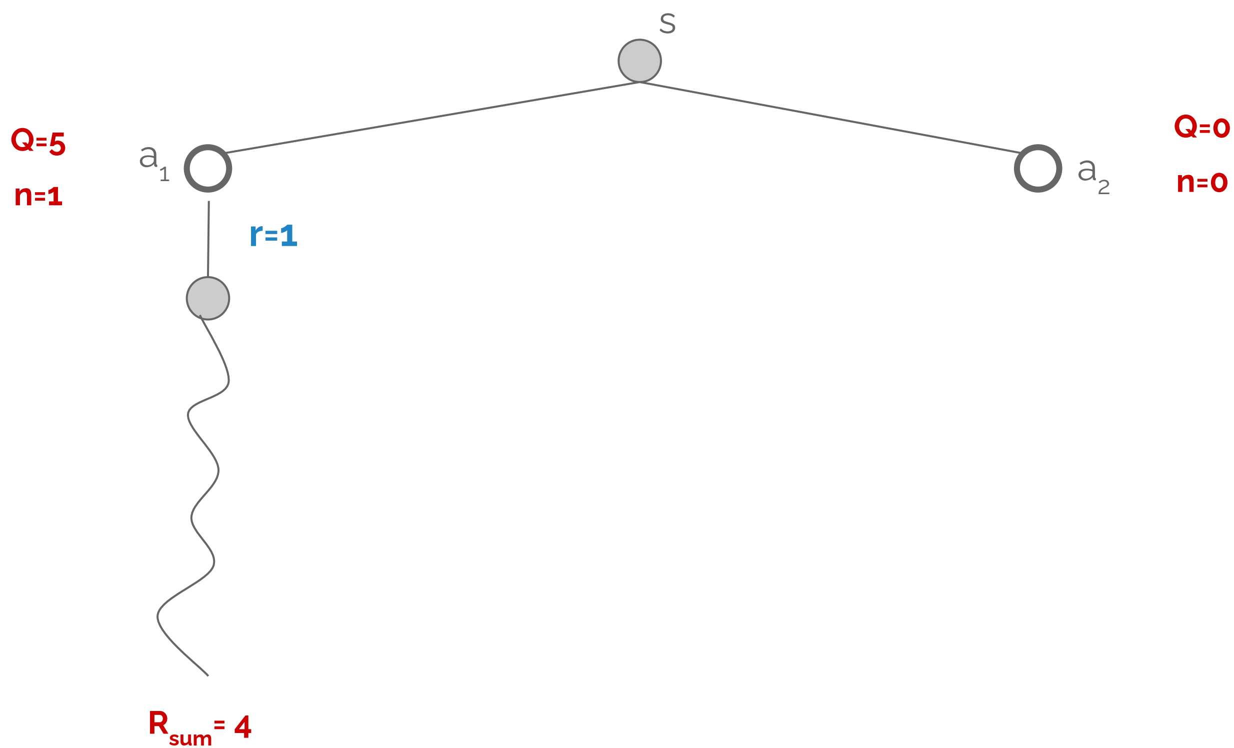
3. Roll-out
4. **Back-up**



Monte Carlo Tree Search

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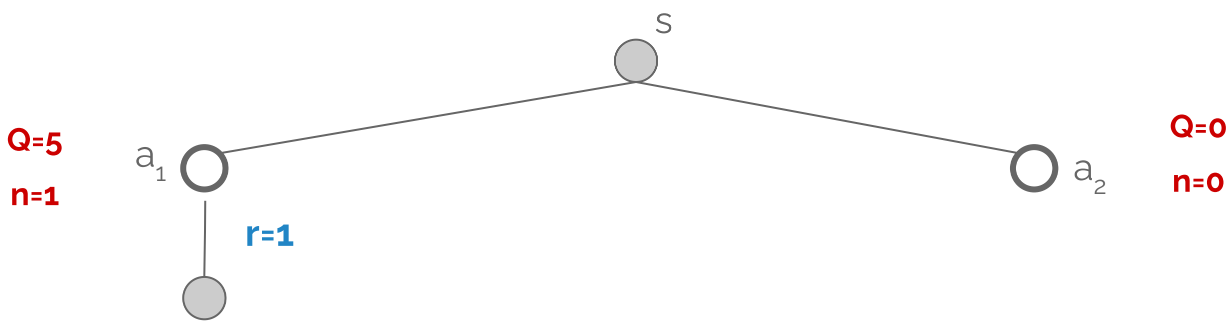
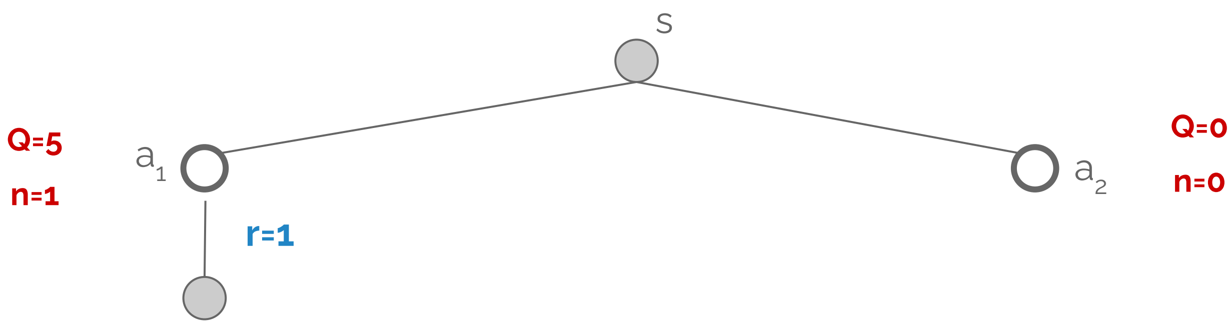


First iteration, repeat!

Monte Carlo Tree Search

1. **Select**
2. Expand
3. Roll-out
4. Back-up



$$\pi_{UCT}(s) = \arg\max_a Q(s,a) + c\sqrt{\frac{\ln n(s)}{n(s,a)}}$$

Monte Carlo Tree Search

1. **Select**
2. Expand
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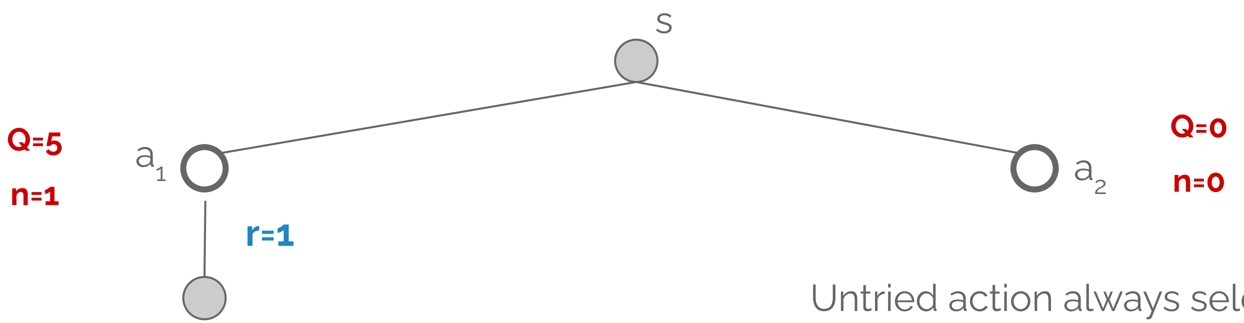
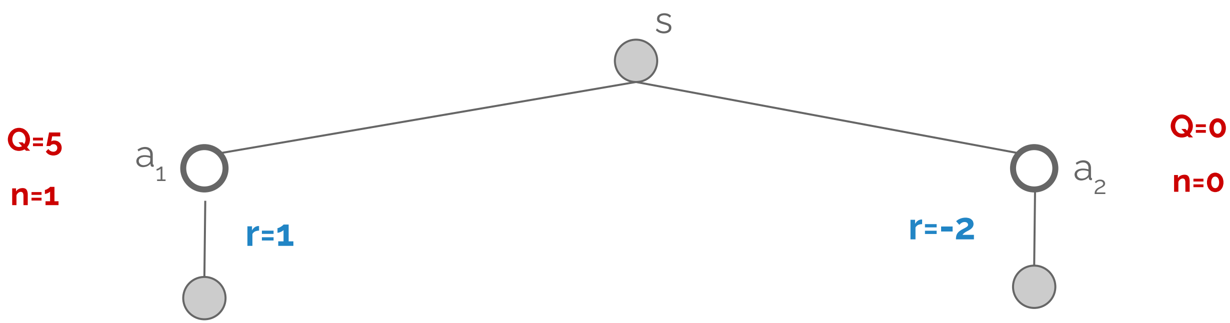


Untried action always selected
(=expand a_2)

$$\pi_{UCT}(s) = \arg\max_a Q(s,a) + c\sqrt{\frac{\ln n(s)}{n(s,a)}}$$

Monte Carlo Tree Search

1. Select
2. **Expand**
3. Roll-out
4. Back-up



Monte Carlo Tree Search

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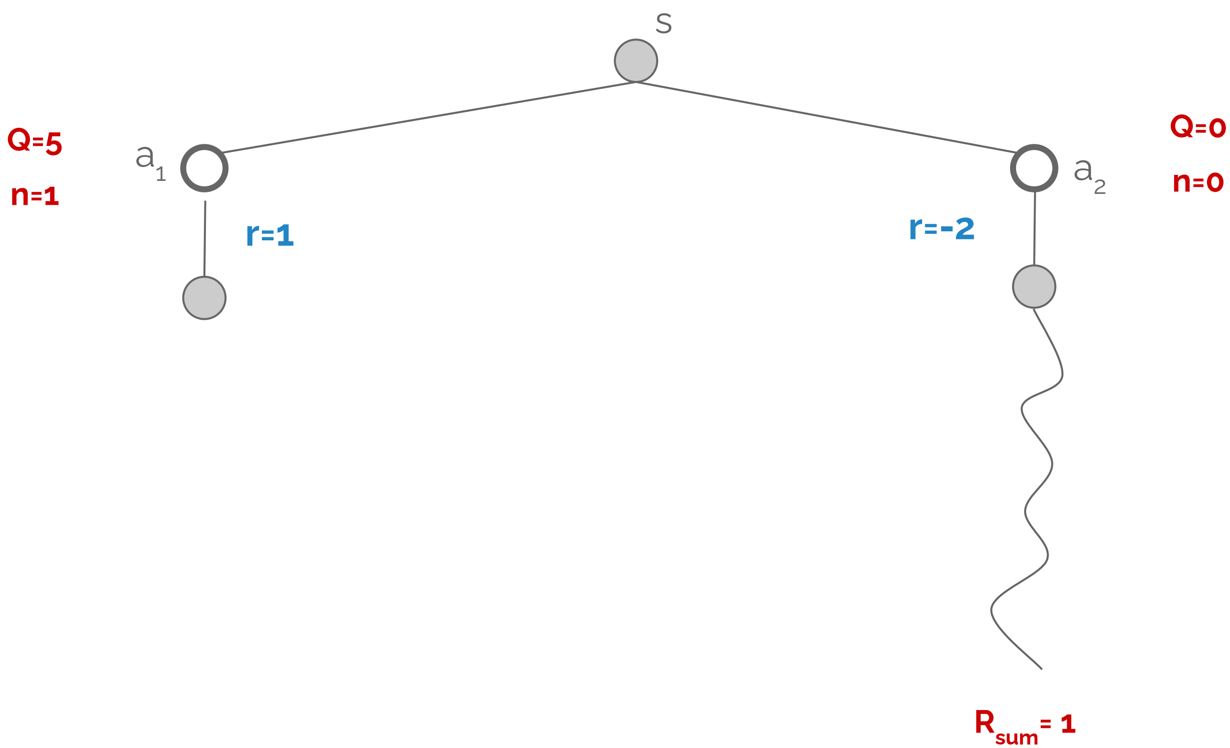
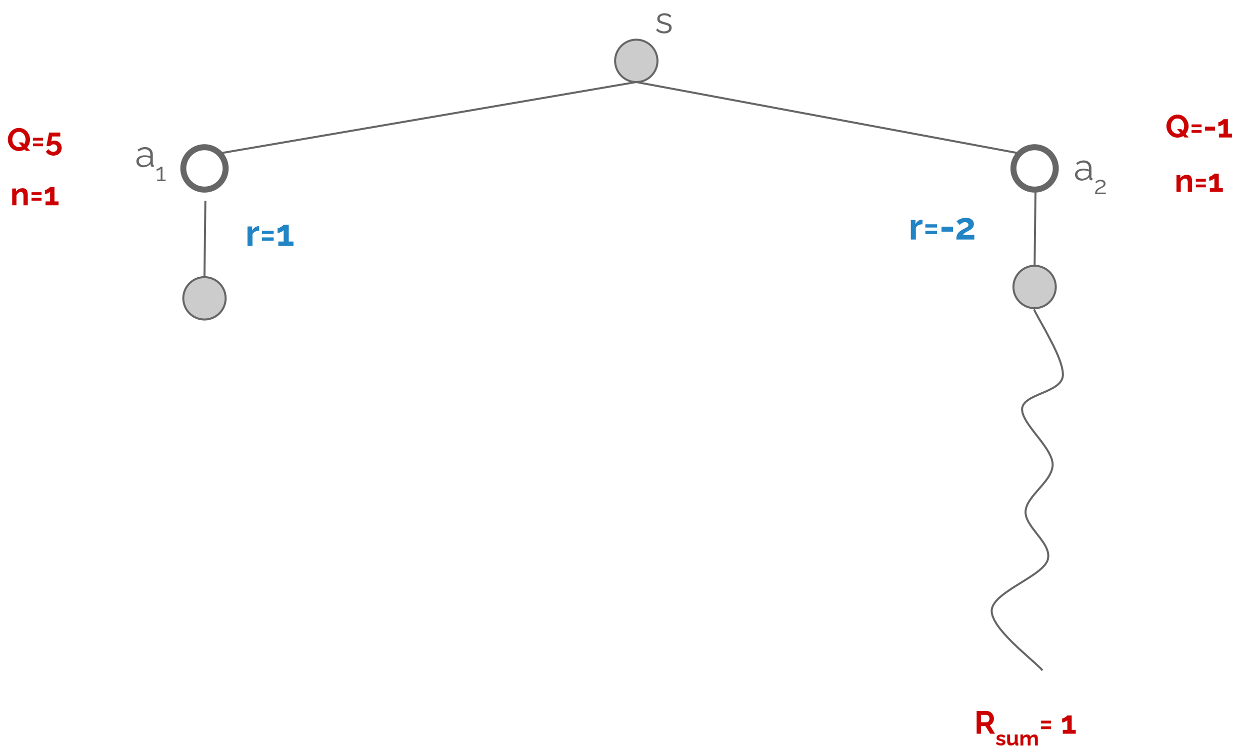
3. **Roll-out**
4. Back-up



Monte Carlo Tree Search

1. Select
2. Expand
3. Roll-out
4. **Back-up**



Monte Carlo Tree Search

1. Select
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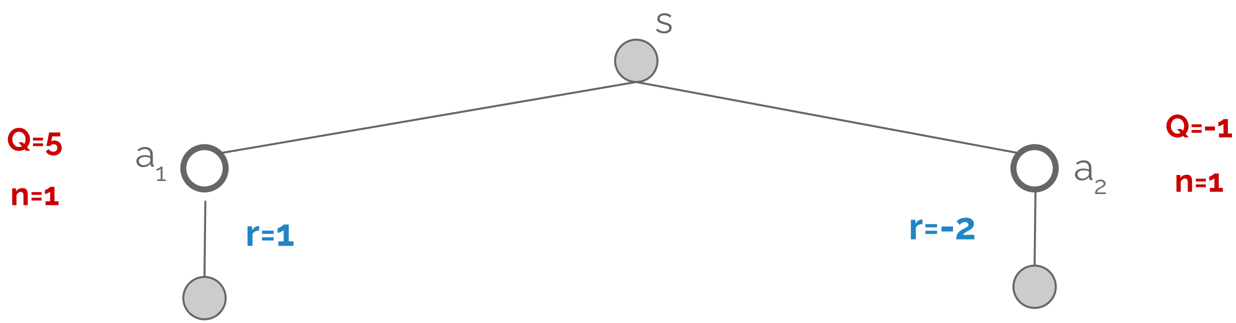
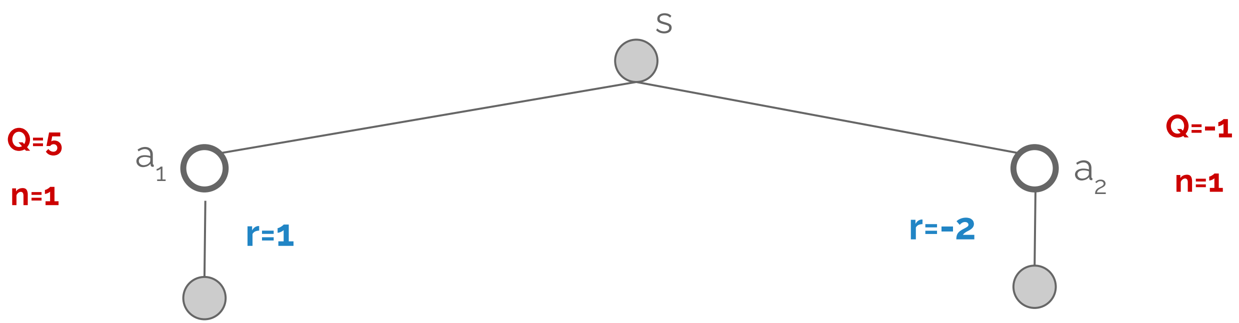
4. Back-up



Second iteration, repeat!

Monte Carlo Tree Search

1. **Select**
2. Expand
3. Roll-out
4. Back-up



Monte Carlo Tree Search

1. **Select**
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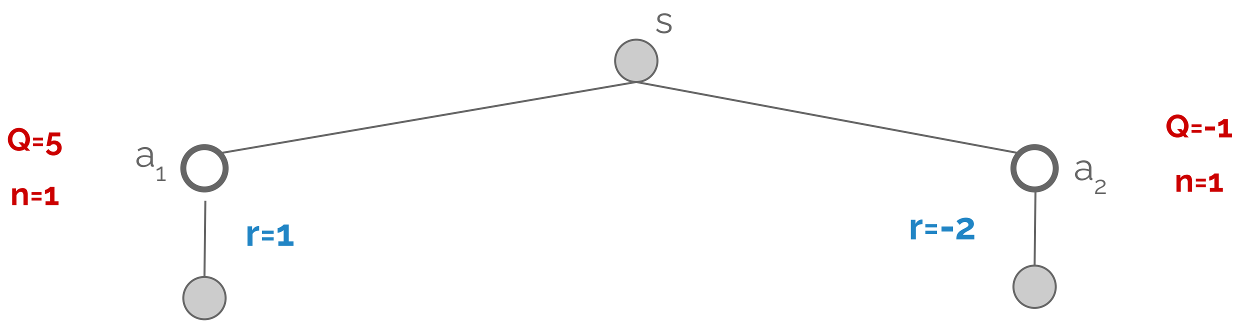

No untried action left, so now we really use the UCT select rule (assume c=1.0)

$$\pi_{UCT}(s) = \arg\max_a Q(s,a) + c\sqrt{\frac{\ln n(s)}{n(s,a)}}$$

Monte Carlo Tree Search

1. **Select**
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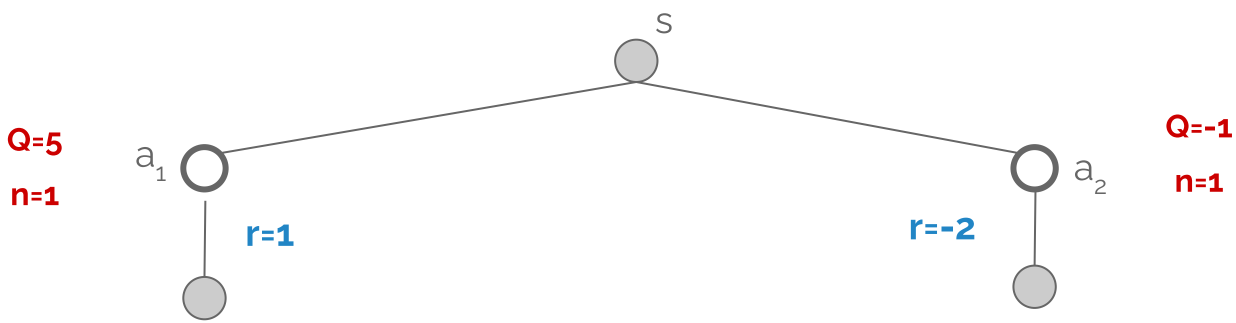


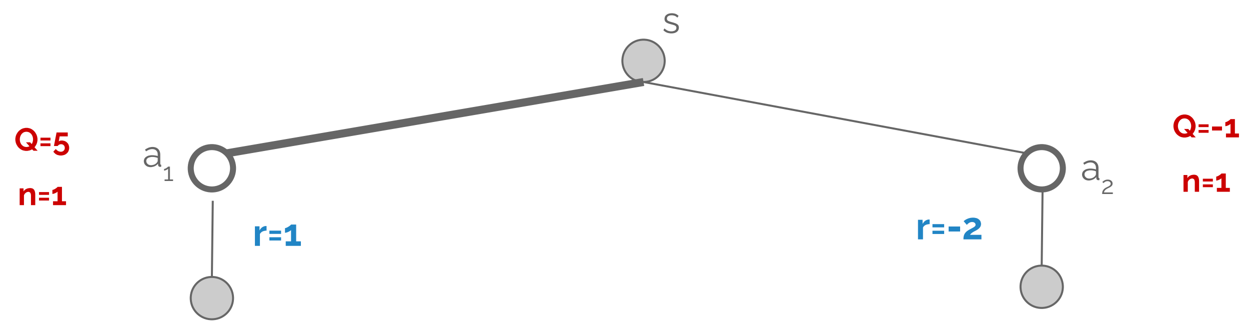
No untried action left, so now we really use the UCT select rule (assume c=1.0)

$$\pi_{UCT}(s) = \arg\max_a Q(s,a) + c\sqrt{\frac{\ln n(s)}{n(s,a)}}$$

Q=5, n(s,a)=1, n(s)=2	Q=-1, n(s,a)=1, n(s)=2
UCT = 5.8	UCT = -0.2

Monte Carlo Tree Search

1. **Select**
2. Expand
3. Roll-out
4. Back-up



No untried action left, so now we really use the UCT select rule (assume c=1.0)

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Q=5, n(s,a)=1, n(s)=2

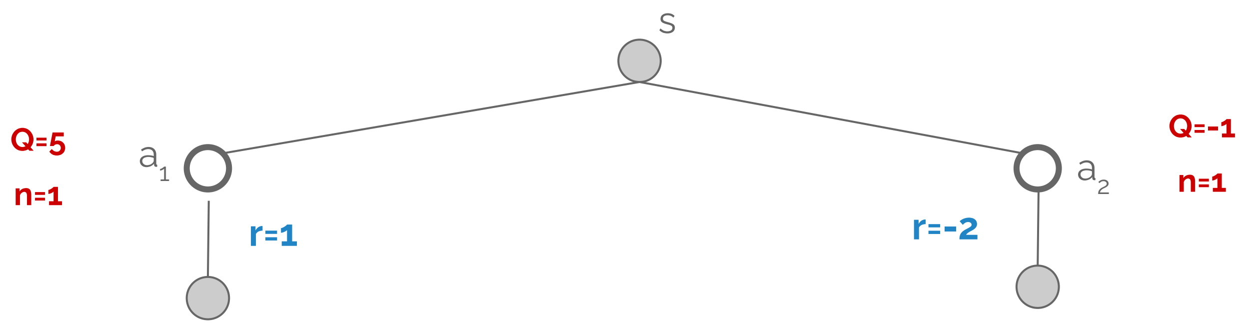
UCT = 5.8

Q=-1, n(s,a)=1, n(s)=2

UCT = -0.2

Monte Carlo Tree Search

1. **Select**
2. Expand
3. Roll-out
4. Back-up

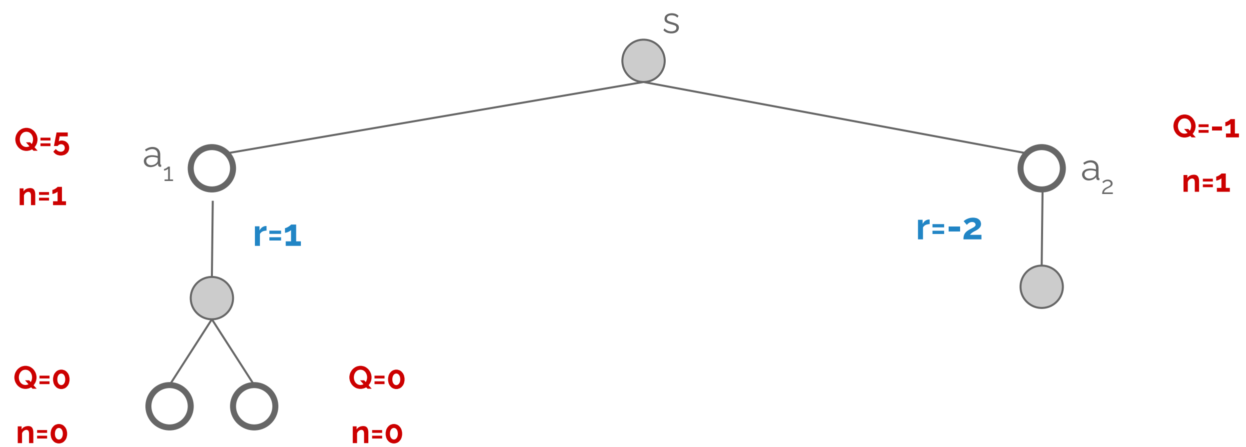


Now we need to select at this state at depth 1.

What will happen?

Monte Carlo Tree Search

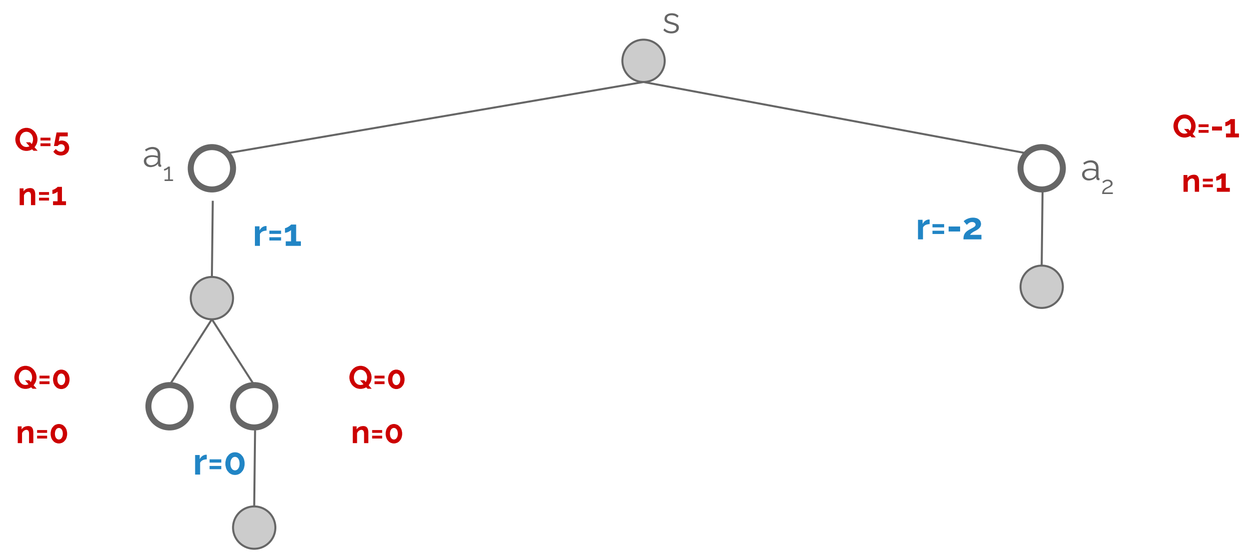
1. **Select**
2. Expand
3. Roll-out
4. Back-up



Unvisited actions, need to expand,
randomly pick one of the available actions

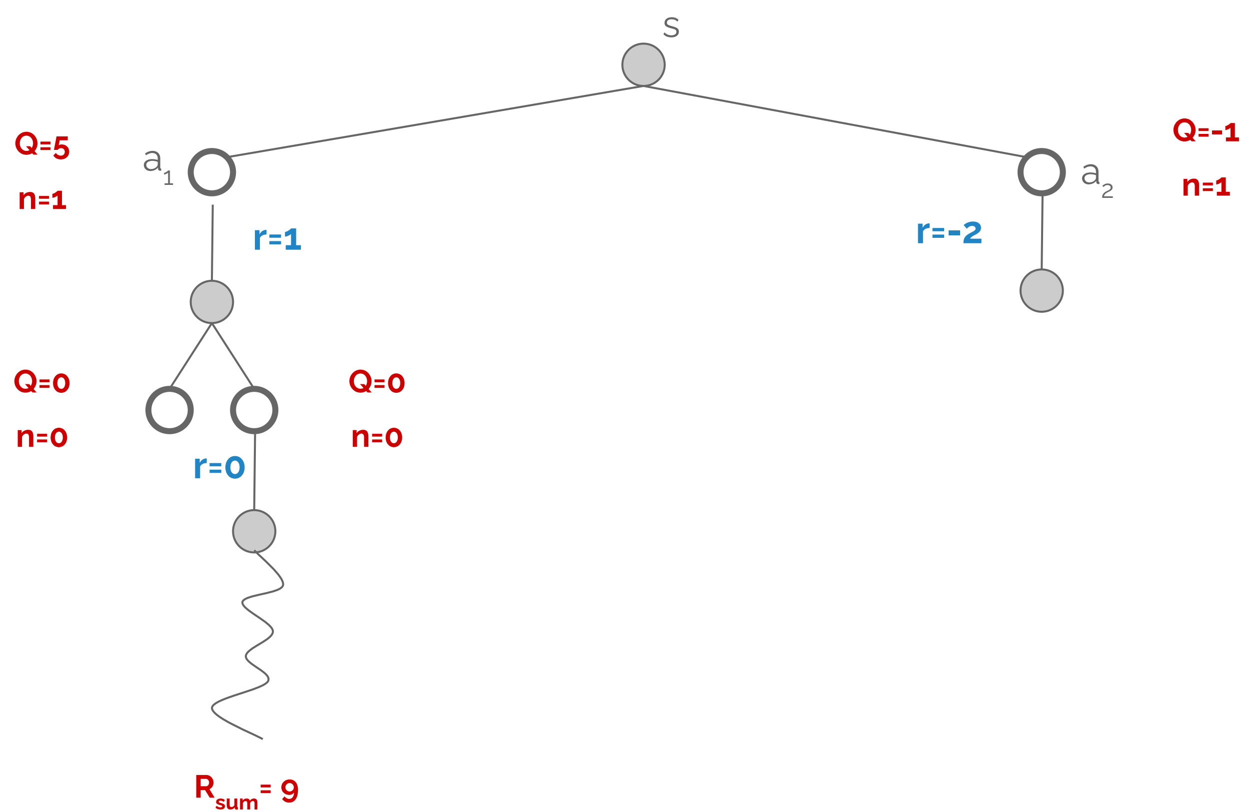
Monte Carlo Tree Search

1. Select
2. **Expand**
3. Roll-out
4. Back-up



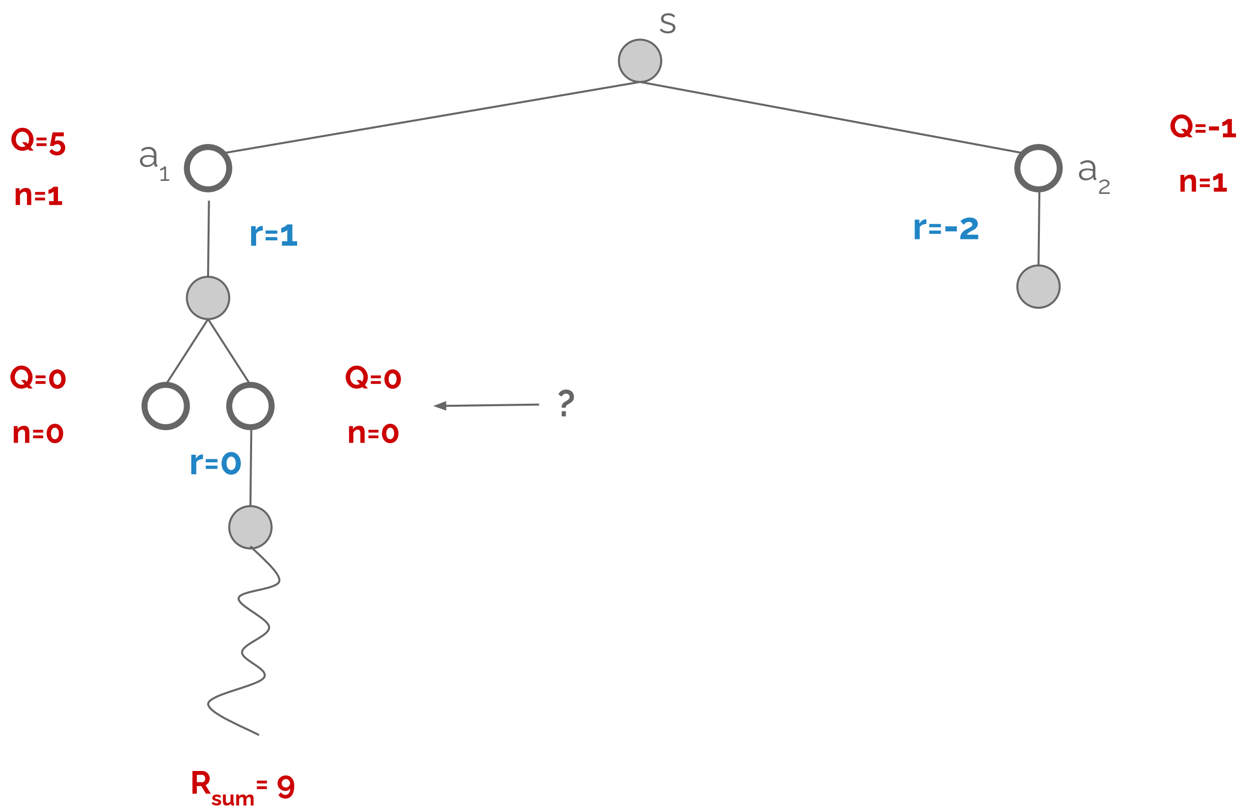
Monte Carlo Tree Search

1. Select
2. Expand
3. **Roll-out**
4. Back-up



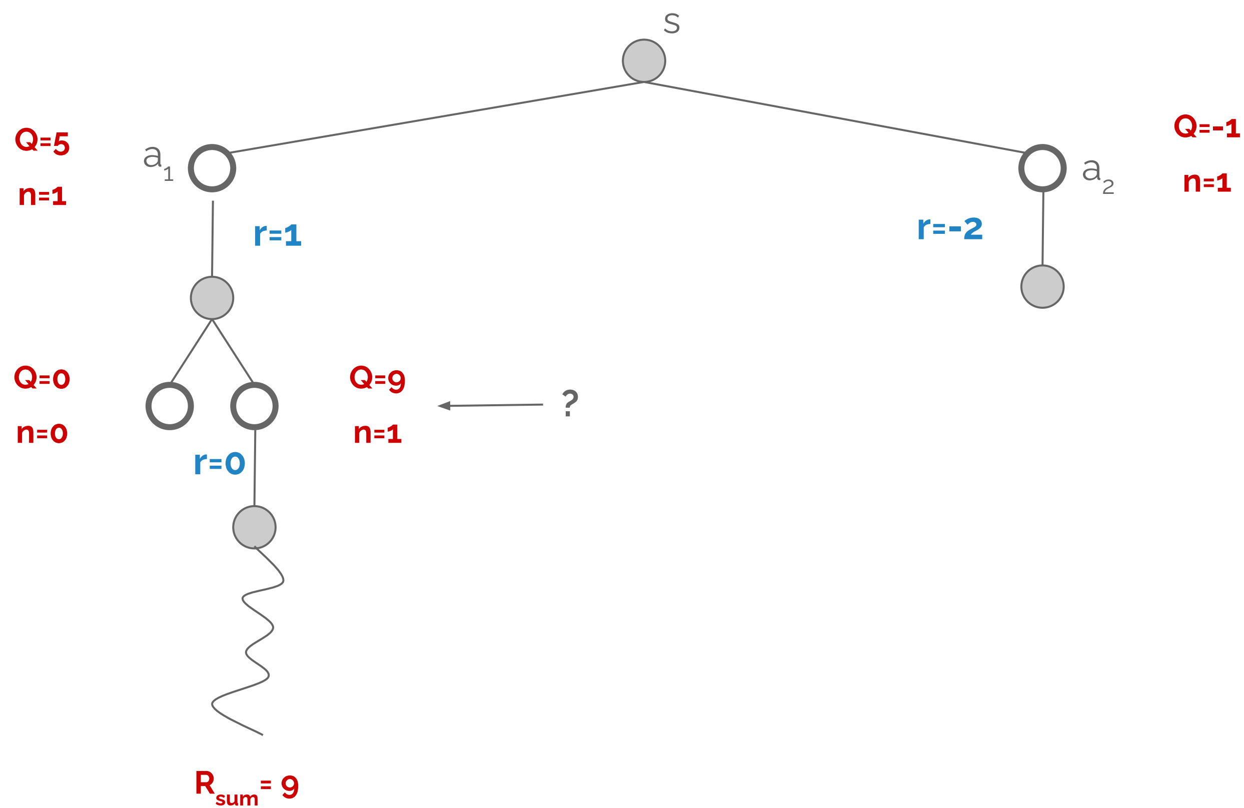
Monte Carlo Tree Search

1. Select
2. Expand
3. Roll-out
4. **Back-up**



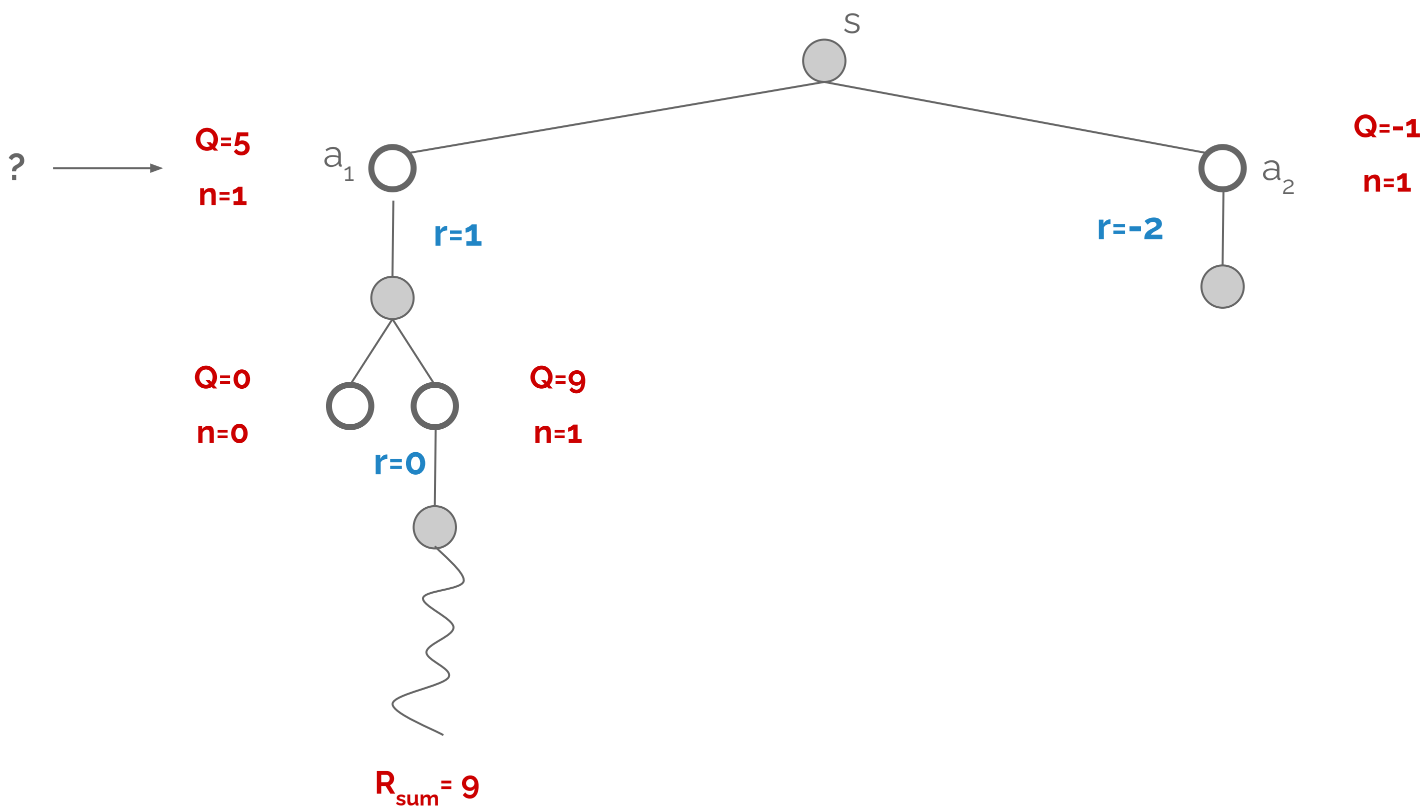
Monte Carlo Tree Search

1. Select
2. Expand
3. Roll-out
4. **Back-up**



Monte Carlo Tree Search

1. Select
2. Expand
3. Roll-out
4. **Back-up**



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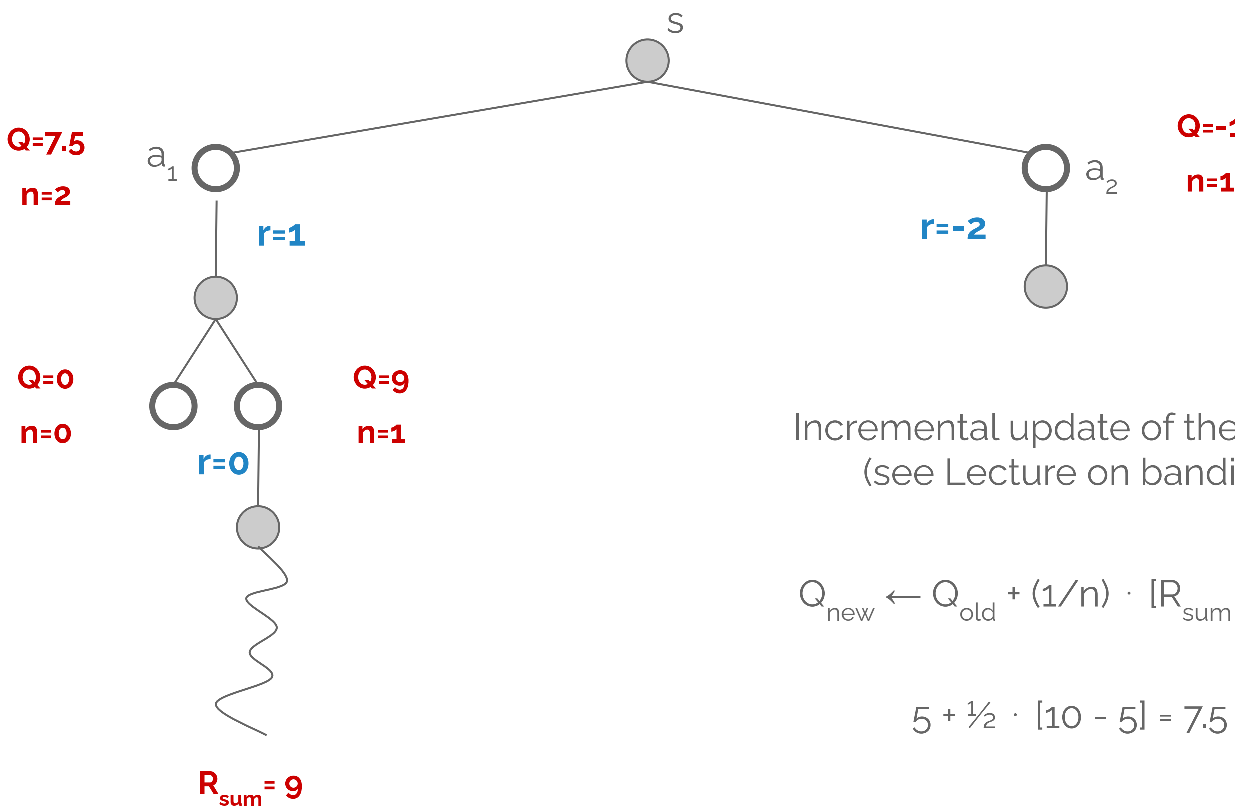


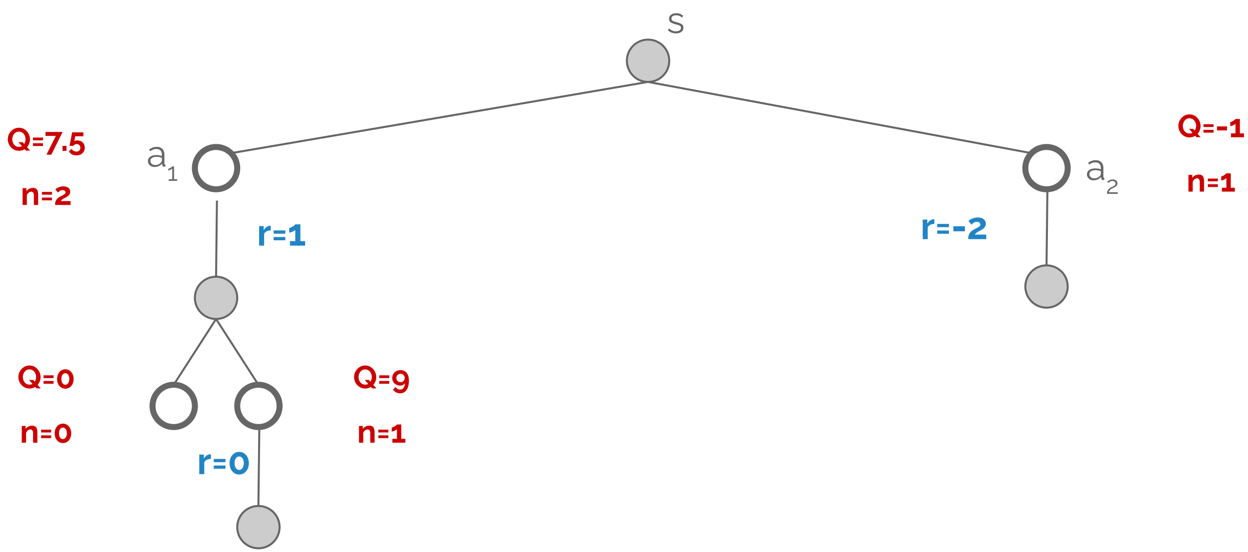
Incremental update of the mean
(see Lecture on bandits)

$$Q_{new} \leftarrow Q_{old} + (1/n) \cdot [R_{sum} - Q_{old}]$$

$$5 + ½ \cdot [10 - 5] = 7.5$$

Monte Carlo Tree Search

1. Select
2. Expand
3. Roll-out
4. Back-up



Third iteration,

repeat until trace budget M is up.

Monte Carlo Tree Search

1. Select
2. Expand
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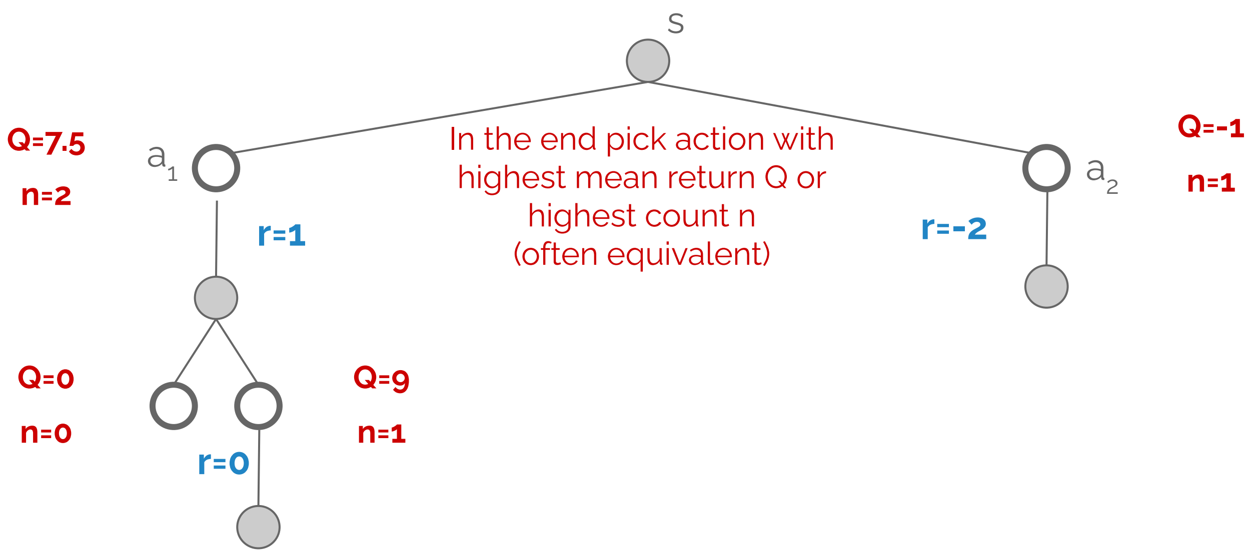
4. Back-up



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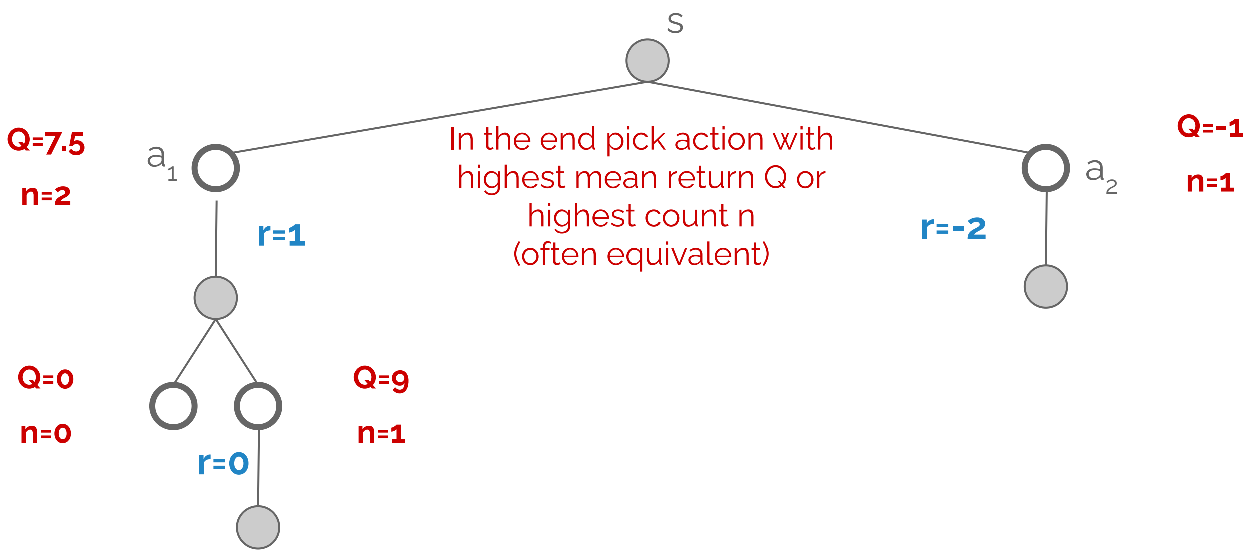

Third iteration,

repeat until trace budget M is up.

Sample complexity: ?

Monte Carlo Tree Search

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2. Expand
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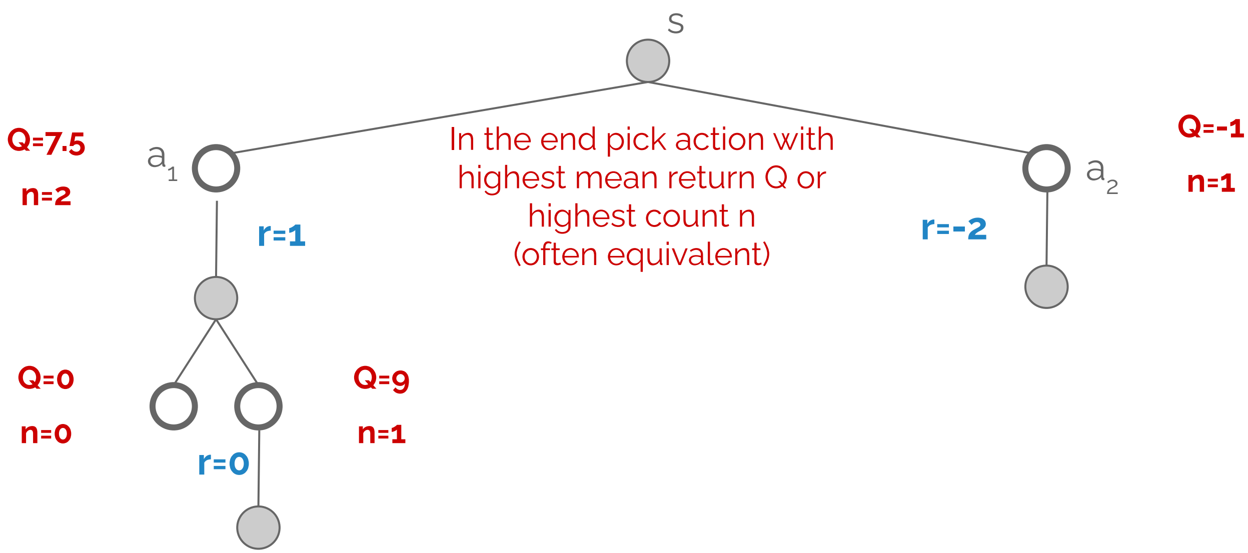
4. Back-up



Third iteration,

repeat until trace budget M is up.

Sample complexity: $M \cdot D$

Monte Carlo Tree Search

1. Select
2. Expand
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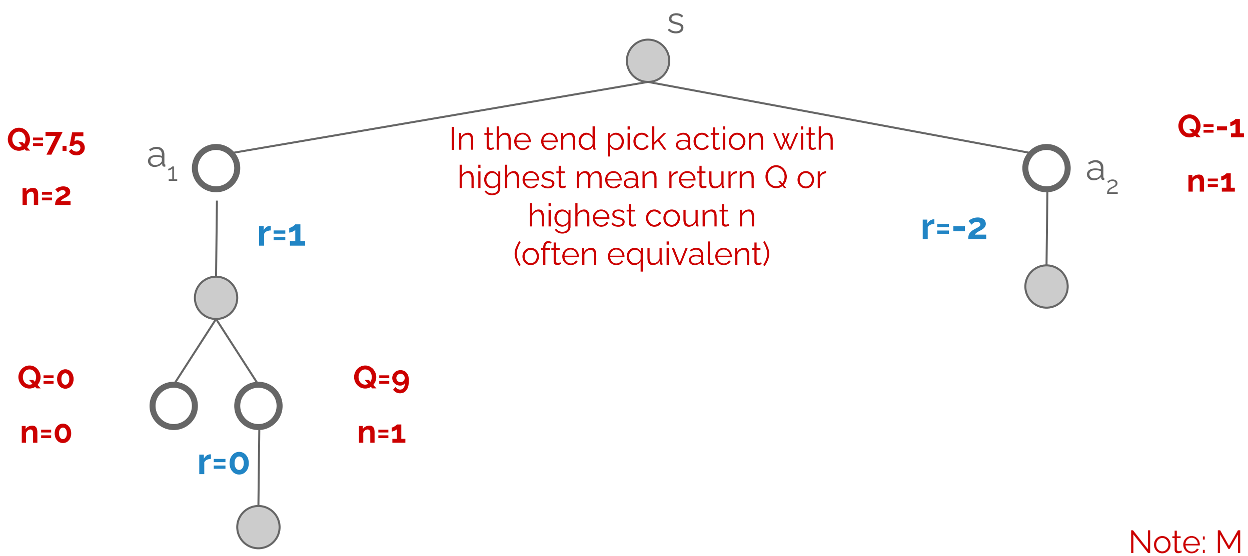

In the end pick action with highest mean return Q or highest count n (often equivalent)

Third iteration,

repeat until trace budget M is up.

Note: M in MCTS (total # of traces) is not the same as N (# of traces per action) in MCS and SS

Sample complexity: $M \cdot D$

Summary: Monte Carlo Tree Search

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- Very powerful search paradigm: adaptively focuses search budget based on statistical uncertainty measures.

Summary: Monte Carlo Tree Search

- Very powerful search paradigm: adaptively focuses search budget based on statistical uncertainty measures.

- One of the most popular algorithms in problems without a good heuristic.

4. Iterated planning & learning

Planning versus learning

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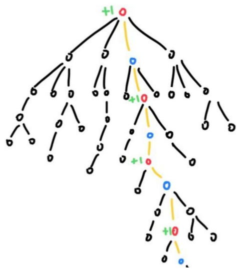
Pure planning is often suboptimal

Planning versus learning

Pure planning is often suboptimal

-

Uninformed (sample-based) search is too expensive & we lack good heuristics



Planning versus learning

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-

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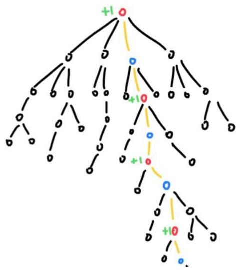

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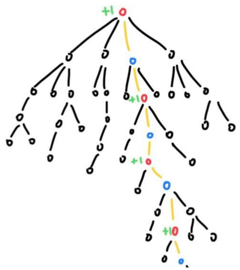
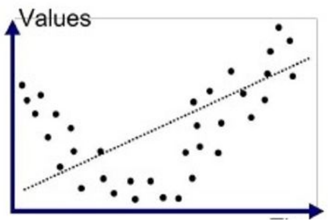


Pure learning is often suboptimal

-

Learned approximate value/policy typically has remaining errors



Planning versus learning

Pure planning is often suboptimal

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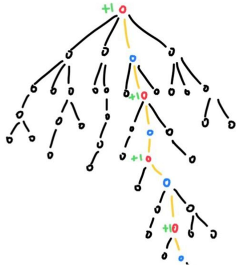

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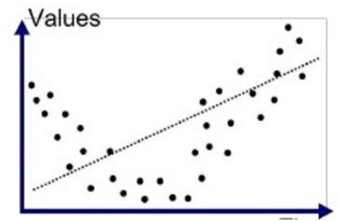
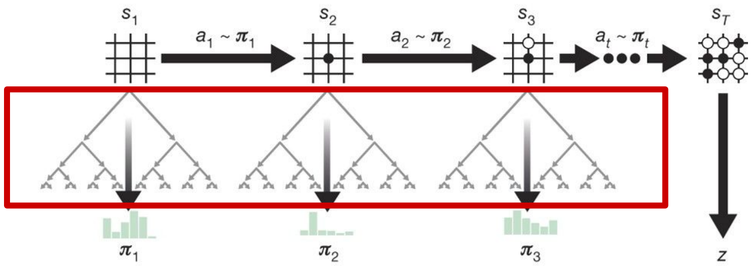


<u>But both approaches can be combined!</u>

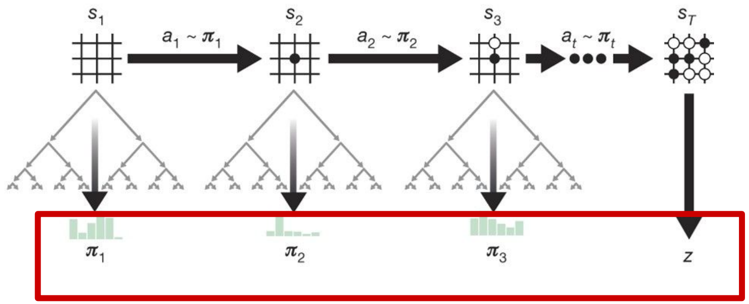
AlphaGo

AlphaGo



Planning..

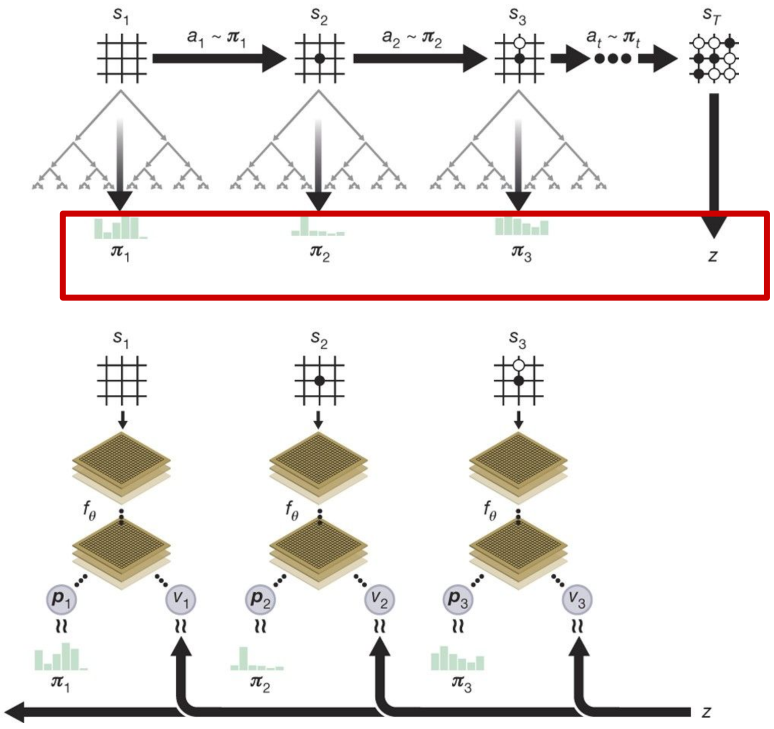
AlphaGo



Planning..

..generates statistics..

AlphaGo



Planning..

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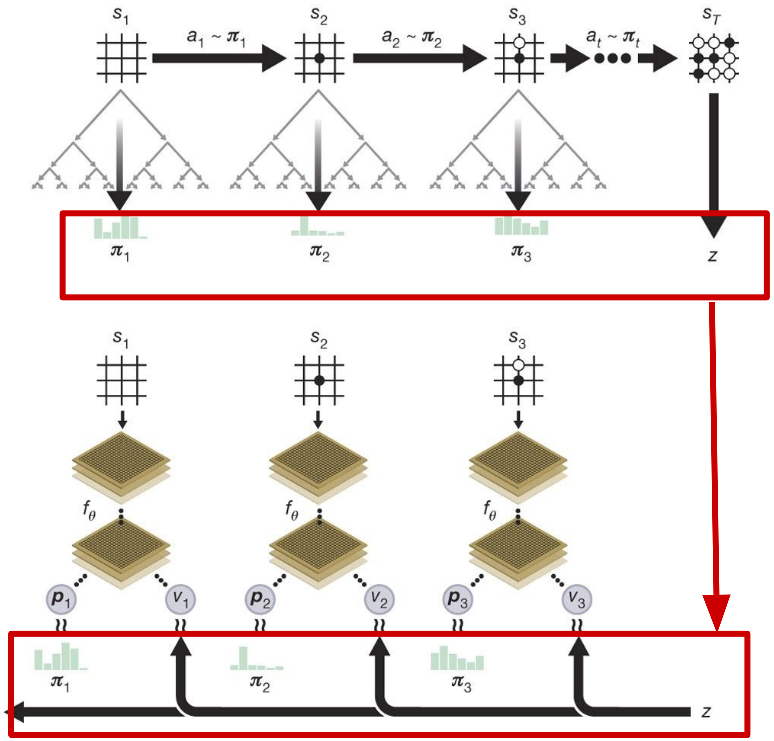
AlphaGo

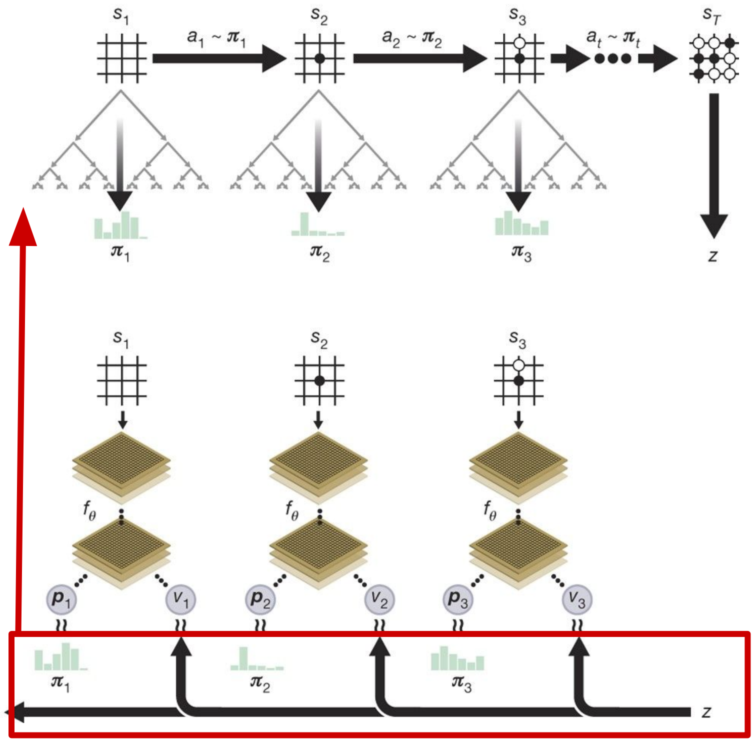


Planning..

..generates statistics..

..that are used to train (approximate) value and policy functions..

AlphaGo

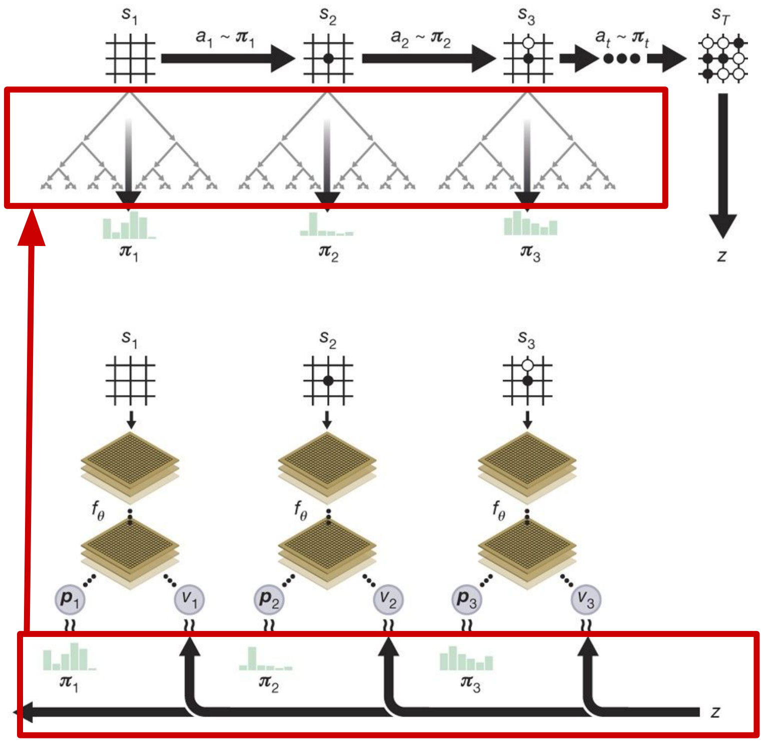


..which we may itself use..

AlphaGo

..to steer new planning iterations.

..which we may itself use..



Iterated planning and learning

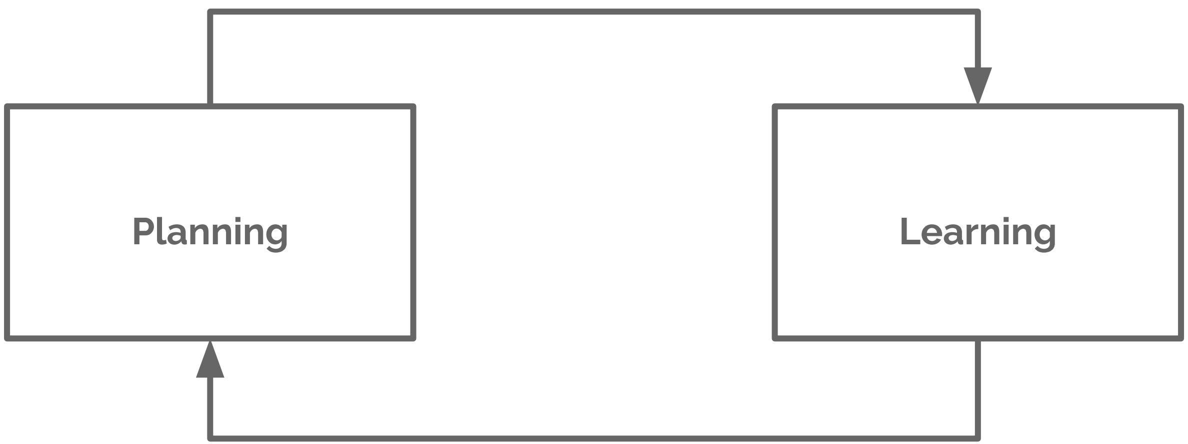
Iterated planning and learning

Iterated planning and learning

… use learned value/policy function to steer new planning iterations.

Iterated planning and learning

.. use planning to **1)** correct errors in learned solution (*'decision-time planning'*) and/or
2) generate training data for learning (*'background planning'*).

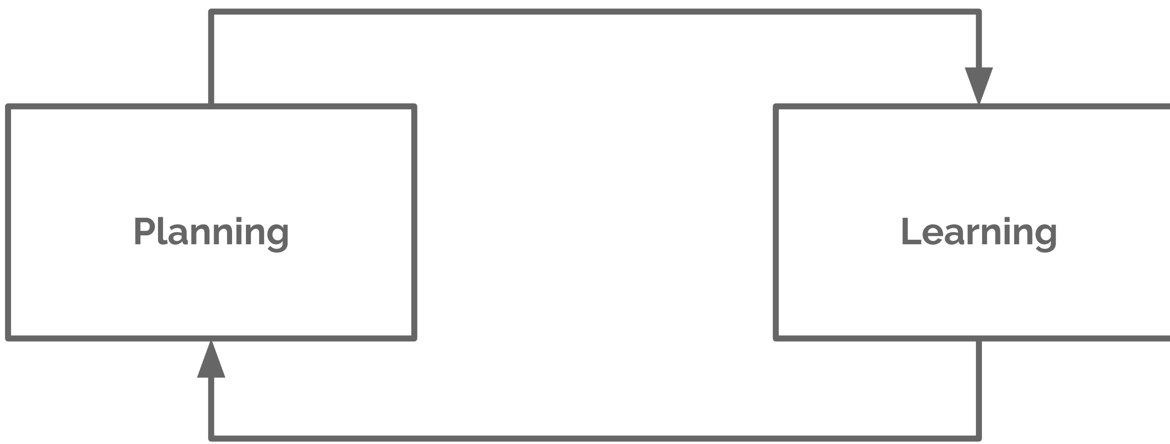


... use learned value/policy function to steer new planning iterations.

Iterated planning and learning

.. use planning to **1)** correct errors in learned solution (*'decision-time planning'*) and/or **2)** generate training data for learning (*'background planning'*).

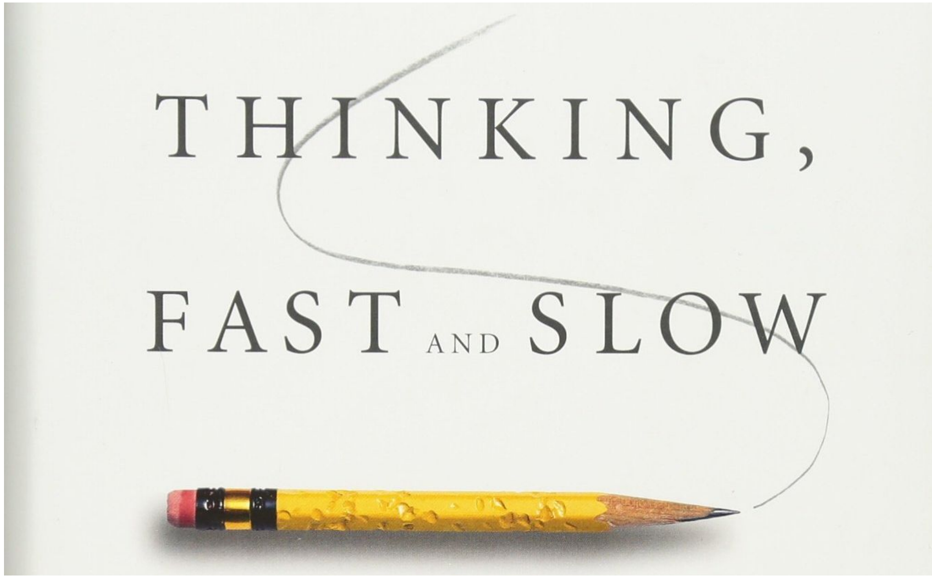
Both types of planning are useful/combined in this iterated scheme



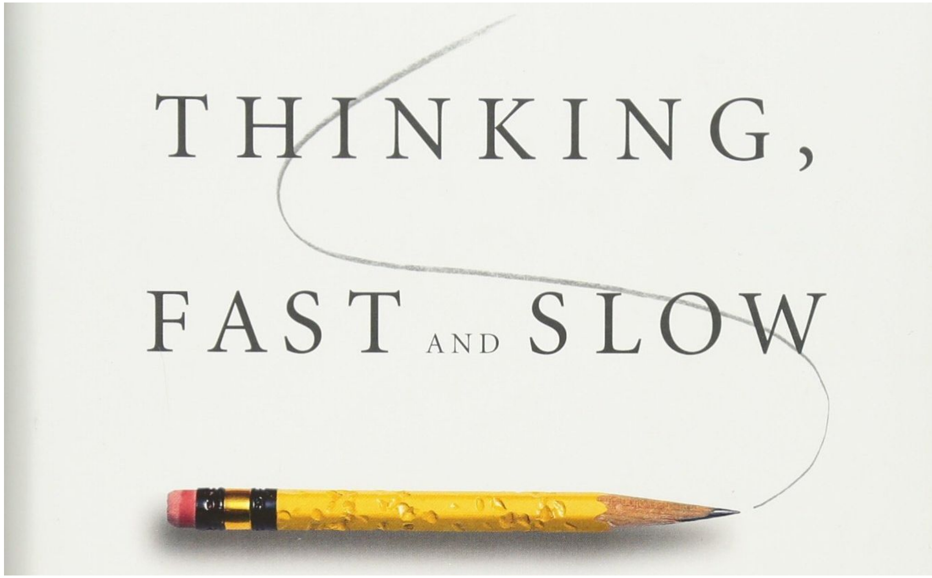
... use learned value/policy function to steer new planning iterations.

Thinking fast & slow

Thinking fast & slow



Thinking fast & slow



Psychology research, but well interpretable in terms of AI

Thinking fast & slow

Thinking fast & slow

Learned (approximate) value function

=

‘Thinking fast’

Thinking fast & slow

Learned (approximate) value function

=

'Thinking fast'

(reactive behaviour based on pattern recognition in known situations)

Thinking fast & slow

Learned (approximate) value function

=

'Thinking fast'

(reactive behaviour based on pattern recognition in known situations)

Decision-time planning

=

'Thinking slow'

Thinking fast & slow

Learned (approximate) value function

=

'Thinking fast'

(reactive behaviour based on pattern recognition in known situations)

Decision-time planning

=

'Thinking slow'

(putting local effort in current decision to overcome errors in the learned value function)

Thinking fast & slow

Learned (approximate) value function

=

'Thinking fast'

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Both have their role in optimal decision-making!

Thinking fast & slow

Learned (approximate) value function

=

'Thinking fast'

(reactive behaviour based on pattern recognition in known situations)

Decision-time planning

=

'Thinking slow'

(putting local effort in current decision to overcome errors in the learned value function)

Both have their role in optimal decision-making!

(more in later lecture on AlphaGo)

Summary

1. Decision-time versus background planning
2. Classic planning
3. Monte Carlo search
4. Iterated planning & learning

Summary

1. Decision-time versus background planning

2. Classic planning

3. Monte Carlo search

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Questions?